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Thermal Regime of the Earth's Outer Core

By W. ULLMANN[1]) and U. WALZER[1])

Summary – First an introduction into the dynamo problem and the core paradox is given. A novel theory of the volume dependence of Grüneisen's parameter is used for calculating the adiabatic temperatures on the assumption that the melting temperature of the material is reached at the boundary between the inner and outer cores of the Earth (IOB). On this condition, thermal convection *throughout* the outer core is impossible according to the melting-point curves of KENNEDY and HIGGINS (1973) and LIU (1975), whereas it is permitted by that of LEPPALUOTO (1972), BOSCHI (1975), and STACEY (1977). Various possibilities of solving the core paradox are described.

Zusammenfassung – Zunächst geben wir eine Einführung in das Dynamo-Problem und das Kernparadoxon. Mit einer neuen Theorie der Volumenabhängigkeit des Grüneisen-Parameters werden unter der Voraussetzung, daß an der Grenze zwischen innerem und äußerem Erdkern (IOB) die Schmelztemperatur des Materials erreicht ist, die adiabatischen Temperaturen berechnet. Für die Schmelzpunkt-Kurven von KENNEDY und HIGGINS (1973) und LIU (1975) ist unter dieser Voraussetzung thermische Konvektion im *gesamten* äußeren Kern unmöglich, für die von LEPPALUOTO (1972), BOSCHI (1975) und STACEY (1977) erlaubt. Verschiedenartige Möglichkeiten, das Kernparadoxon zu lösen, werden beschrieben.

Key words: Thermal convection; Outer core; Grüneisen's parameter.

1. Introduction: The dynamo problem

At present, the generation of the geomagnetic main field is chiefly explained by a dynamo mechanism. There are a number of ingenious and, in part, rather complicated theories (see, for instance, KRAUSE and RÄDLER, 1971; SCHMUTZER, 1974/75). Comprehensive surveys were given by ROBERTS (1971), GUBBINS (1974), MOFFATT (1976), and STIX (1977). It is now evident that there is an essential relationship between the angular velocity of the planets and the planetary magnetic field, as the Alfvén-to-angular-velocity ratio for Mercury, Venus, Earth, Jupiter, and Saturn is of one and the same order of magnitude (BUSSE, 1976; STIX, 1977). This observation serves as a basis, for instance, for a theory of BUSSE (1975) in which the dominant effect of rotation is expressed by the fact that the Lorentz force is small as compared with the Coriolis force, and the full hydromagnetic problem for a cylindrical configuration is solved.

[1]) Zentralinstitut für Physik der Erde, Institutsteil Jena, 69 Jena, Burgweg 11, DDR; Mitt. d. ZIPE Nr. 682.

As is well known, each dynamo must satisfy two fundamental theorems:

(1) COWLING's (1933) theorem: the magnetic field generated by the dynamo must be neither axisymmetric nor two-dimensional (LORTZ, 1968).
(2) ELSASSER's (1946) theorem: mass flow connected with the dynamo must not be purely toroidal, i.e., the radial component of the velocity field must not vanish.

At a first approximation, planetary magnetic fields are axisymmetric. This, initially, led to the conclusion that dynamos cannot be the source of the terrestrial magnetic field. Now it is known that minor deviations from axial symmetry render dynamos possible (BRAGINSKII, 1964). LORTZ (1972) proved that there are stationary dynamos which are exactly axisymmetric outside the conductor. From this it follows that, even if the magnetic field on the surface of Earth is completely axisymmetric, a dynamo might well be the source of it. The second theorem leads us directly to the main problem of this paper: to obtain a stationary radial velocity component of flows within the outer core, thermal convection is usually assumed to exist there. HIGGINS and KENNEDY (1971), however, claimed that the liquid outer core is stably stratified and thermal convection, therefore, should be impossible. This claim is based on an extrapolation of the melting-point curve of KRAUT and KENNEDY (1966) which shows a smaller gradient with depth than the curve of adiabatic temperature. If we assume that the outer and inner cores are of the same chemical composition, and that the temperature at the boundary between the liquid outer core and the solid inner core corresponds to the melting point of the material at the pressure existing there, it follows that the adiabatic temperature required for thermal convection in the outer core is below the melting temperature, demanding that the outer core be solid, instead of liquid, as observed. This core paradox has been discussed and criticized by various authors and is no longer regarded as a serious problem (for the more relevant melting-point curves). However, it stimulated more careful studies of the thermal regime of the core. In this paper we present a new theory of adiabatic temperature, different versions of the pressure dependence of the melting temperature of iron, and other things to contribute to the elucidation of these fundamental questions.

Another way to drive the dynamo is gravitational differentiation of the outer core matter (LOPER and ROBERTS, 1978; HÄGE and MÜLLER, 1979). These interesting approaches are not discussed in this paper.

2. *Origin and parameterization of basic relations*

2.1. *Some basic aspects*

For our purpose a spherically symmetric model of the Earth's outer core, as presented by DZIEWONSKI, HALES and LAPWOOD (1975), is adequate. We consider a homogeneous thermodynamic system with definite boundaries, that is, a phase which

may contain more than one component, as is most probably the case in the outer core. Moreover, the system is in stable equilibrium, and it has two degrees of freedom. Thus the state of the system is uniquely determined by two independent thermodynamic variables.

We suppose a portion of the outer core at any depth z to be represented by such a system. As z can be chosen arbitrarily we consider the parameters of state of that system, used to describe and correlate local properties of the outer core materials in bulk, to be functions of z over the depth interval from 2885.3 km to 5153.9 km. These functions are strictly monotonic and continuous, because phase changes are not expected anywhere. Hence it follows that the thermodynamic system may undergo a series of state changes which are reversible solely, and we can now use general methods of equilibrium thermodynamics.

To form the non-dimensional quantity

$$-\frac{V}{T}\left(\frac{\partial T}{\partial V}\right)_S \equiv \gamma, \tag{1}$$

temperature T is regarded as a function of volume V and entropy S. γ is often called the (thermal) Grüneisen parameter. It is equivalent to the parameter originally defined by Grüneisen (logarithmic dependence of a lattice frequency upon volume) only if it is independent of temperature, a restriction that is questionable in the case of the Earth's outer core, as was pointed out by STACEY (1977) and others. However, γ as defined by (1), is the important quantity for practical purposes.

The phase of the outer core to which our attention is confined may be considered as a two-component system consisting of a nearly close-packed atomic arrangement, apparently metallized in spite of being completely dislocated in the fluid state, and an electronic fluid (conduction electrons). If the system thus defined is supposed to be like a mixture of two perfect, unreacting gases, then the pressure P, entropy S, and thermodynamic potentials of the system are all sums of the corresponding quantities which either component would have if present alone.

Hence, in particular, it follows that the total heat capacity at constant volume denoted by C_V results from

$$C_V = C_V^{(1)} + C_V^{(2)}, \tag{2}$$

where the heat capacities $C_V^{(1)}$ and $C_V^{(2)}$ of the same kind refer to the lattice and the electronic fluid, respectively. The Grüneisen parameter (1) may be expressed by

$$\gamma = \frac{\gamma^{(1)} C_V^{(1)} + \gamma^{(2)} C_V^{(2)}}{C_V^{(1)} + C_V^{(2)}}, \tag{3}$$

where $\gamma^{(1)}$ and $\gamma^{(2)}$ are the lattice and electron contributions to (1). It is to be noted that, although both $\gamma^{(1)}$ and $\gamma^{(2)}$ may be taken to be independent of temperature, since $C_V^{(1)}$ and $C_V^{(2)}$ are different functions of temperature, γ is independent of temperature only if $\gamma^{(1)} = \gamma^{(2)}$.

Thermodynamic relationships lead to

$$\left(\frac{\partial\gamma}{\partial \ln T}\right)_V = \left(\frac{\partial \ln C_V}{\partial \ln V}\right)_S = \left(\frac{\partial \ln C_V}{\partial \ln V}\right)_T - \gamma\left(\frac{\partial \ln C_V}{\partial \ln T}\right)_V \tag{4}$$

(see also BRENNAN and STACEY, 1979). From this, considering a high-temperature classical situation in which $C_V^{(1)}$ suffices to be taken as a constant, independent of both T and V, $\gamma^{(1)}$ cannot depend upon temperature in the outer core.

According to the theory of the electronic spectrum of metals which was presented for the first time by BLOCH in 1929 (see, e.g., LANDAU and LIFSCHITZ, 1966), the electron contribution to the heat capacity C_V, that is, $C_V^{(2)}$ must be written as proportional to temperature T. Thus,

$$(\partial \ln C_V^{(2)}/\partial \ln T)_V = 1 \tag{5}$$

and taking $\gamma^{(2)}$ also independent of T, from (4) and (5) it is given by

$$\gamma^{(2)} = \partial \ln C_V^{(2)}(V, T)/\partial \ln V. \tag{6}$$

Thus both $\gamma^{(1)}$ and $\gamma^{(2)}$ are functions of V only. From (6) we see that $\gamma^{(2)}$ is determined if $C_V^{(2)}(V, T_2)$ is known and conversely. According to STACEY (1977) we may write

$$C_V^{(2)}(V, T_2) = C_V^{(2)}(V_0, T_2)x^{2/3} \exp\left[-(3\gamma_0^{(2)} - 2)(x^{-1/3} - 1)\right], \tag{7}$$

where V_0 and $\gamma_0^{(2)}$ are the zero-pressure values of V and $\gamma^{(2)}$ (at $T = T_2$), respectively (for the high pressure phase), and use is made of the notation $x \equiv V/V_0$ for the volumetric contraction ($x < 1$). Hence, from (6)

$$\gamma^{(2)} = \tfrac{2}{3} + (\gamma_0^{(2)} - \tfrac{2}{3})x^{-1/3}. \tag{8}$$

The complete determination of $\gamma^{(2)}$ thus requires information on the values of V_0 and $\gamma_0^{(2)}$ from other investigations. Since in the case of $\gamma^{(1)}$ a relation of the form (6) is not available, additional considerations are required. From the diversity of expressions for the Grüneisen parameter derived by many investigators from quite different considerations we have chosen the VASHCHENKO-ZUBAREV (1963) formulation (see also IRVINE and STACEY, 1975; STACEY, 1977), that is, we then write

$$\gamma = \frac{\frac{1}{2}\left(\frac{\partial\kappa}{\partial P}\right)_S - \frac{5}{6} + \frac{2}{9}\frac{P}{\kappa}}{1 - \frac{4}{3}\frac{P}{\kappa}} \tag{9}$$

where all parameters are total (lattice plus electron) contributions for the material. We may also write this equation with the superscript 1 in brackets with each parameter to indicate that the respective quantity refers to the lattice in the outer core and $\kappa^{(1)}$ denotes the corresponding adiabatic bulk modulus, $\kappa^{(1)} \equiv -V(\partial P^{(1)}/\partial V)_{S^{(1)}}$. In this case the outer core is supposed to consist of a material in which interatomic forces can

be considered central as a good approximation and, strictly speaking, the quantities $P^{(1)}$, $\kappa^{(1)}$ and $(\partial\kappa^{(1)}/\partial P^{(1)})_{S^{(1)}}$ must be corresponding zero-temperature extrapolations.

The value of $\gamma^{(1)}$ can be estimated separately if the outer-core estimate by the Vashchenko-Zubarev formulation of γ should include only a slight electron contribution, so that the correction may not be larger than the uncertainty in that estimate, then P, $\kappa_S \equiv -V(\partial P/\partial V)_S$ and $(\partial\kappa_S/\partial P)_S$ might be substituted for the corresponding quantities in (9). Accordingly, to compute the function $\gamma^{(1)}(z)$ for several depths through formula (9), values of the seismic compressional wave velocity $v_p(z)$ and density $\rho(z)$ and pressure P may be used from the parametric earth models by DZIEWONSKI *et al.* (1975). Values of $\gamma^{(1)}(z)$ obtained in this way are listed in Table 2.

Instead of applying the model data to formula (9), an explicit expression for the equation of state $P = -(\partial U/\partial V)_S$, where U is the internal energy, and, being derived from it, expressions of κ_S and $(\partial\kappa_S/\partial P)_S$ can be useful for estimation of the material parameters which appear in these approximation functions. In Subsection 2.4 we shall deal with this question. It should be mentioned that, strictly speaking, the two procedures for the utilization of formula (9) do not refer in the last analysis to the foregoing outer-core phase interpretation as a two-component system.

As to $C_V^{(1)}$ being virtually constant, it suffices to take the lattice specific heat $c_V^{(1)}$ to be $3R/m$, where $R \approx 8.317$ [J mol^{-1} K^{-1}] is the molar gas constant, and m is the molar mass for the outer core usually estimated to be 0.05 [kg mol^{-1}] on an average. Hence, $c_V^{(1)} \approx 499$ [J kg^{-1} K^{-1}]. With this we conclude our discussion of the determination of γ through equation (3), remembering the inconsistent constraint we need for the practical use of the approximate relation (9). For a solution of this problem reference may be made to STACEY (1977). The approach preferred by him differs, however, in some respects from that which has been indicated here.

2.2. *A semi-empirical law of γ for the outer core*

The conventional definition of the thermodynamic γ, required for example by the identity (1) is

$$\gamma = \frac{\alpha\kappa_S V}{C_P}, \tag{10}$$

where $\alpha \equiv 1/V(\partial V/\partial T)_P$ is the isobaric cubic expansion, and C_P is the heat capacity at constant pressure. Since $C_P/V = c_P\rho$, where c_P is the specific heat (at constant pressure), and $\kappa_S = v_p^2\rho$ in the case of the fluid outer core, equation (10) can be rewritten as $\gamma = (\alpha/c_P)v_p^2$, which is more suitable for our purpose. It is obvious that each quantity in this formula may be taken as a well behaved function of z. Hence

$$\gamma(z) = \frac{\alpha(z)}{c_P(z)}[v_p(z)]^2. \tag{11}$$

Using $v_p(z)$ according to the outer-core model by DZIEWONSKI *et al.* (1975), for determination of $\gamma(z)$ attention must now be confined to the term $\alpha(z)/c_P(z)$.

To this end we return to the above two-component system representing the outer-core phase. Then, denoting the principal heat capacity by C and $C^{(i)}$ $(i = 1, 2)$, respectively, it can be shown that the quantity

$$\Psi \equiv \frac{C}{\alpha} - \sum_i \frac{C^{(i)}}{\alpha^{(i)}}, \tag{12}$$

where $\alpha^{(i)}$ is the cubic expansion of the ith component, does not depend upon the path of temperature change prescribed by the constraint $V = \text{const}$ or, alternatively, $P = \text{const}$.

From the Grüneisen law we know that within the limits of validity of the theorem of corresponding states the ratio of the heat capacity and the isobaric cubic expansion of a solid is independent of temperature in the classical regime, i.e. at high temperatures. In addition to this we remember the theory dealing with an electronic fluid in a metal subjected to high temperature through which that ratio, now referring to the electrons, proves also to be temperature independent (see LANDAU and LIFSCHITZ, 1966). As a result we consider both $C^{(1)}/\alpha^{(1)}$ and $C^{(2)}/\alpha^{(2)}$ to be independent of outer-core temperature. Then from (12) it follows that even C/α does not depend on T if, and only if, this applies to Ψ.

Let us suppose Ψ to be a function of density ρ solely, at sufficiently high temperature T. We write

$$\left(\frac{c_P}{\alpha}\right)_{T,\rho} = \left(\frac{c_P}{\alpha}\right)_{T,\rho=\bar{\rho}} + \left(\frac{\partial}{\partial\rho}\frac{c_P}{\alpha}\right)_{T,\rho=\bar{\rho}} (\rho - \bar{\rho}) + \cdots, \tag{13}$$

where $\bar{\rho}$ is, for instance, the mean density of the outer core, and

$$\left(\frac{\partial}{\partial T}\frac{c_P}{\alpha}\right)_{T,\rho=\bar{\rho}} = 0, \qquad \left(\frac{\partial^2}{\partial T\,\partial\rho}\frac{c_P}{\alpha}\right)_{T,\rho=\bar{\rho}} = 0, \ldots \tag{14}$$

may be taken for granted, whichever T is chosen from the temperature range of the outer core. But just the constraints (14) remain to be proved.

As we cannot furnish proof exactly, our attention is now confined to the problem of fitting the power series (13) to a set of data from ρ, α and c_P estimations for the outer core which are assumed to approximately satisfy this expansion of c_P/α. From among such data we prefer those given by STACEY (1977) and, as to ρ, by DZIEWONSKI *et al.* (1975). They are specified in Table 1. For computational convenience, however, equation (13) is rewritten as

$$\frac{c_P(z)}{\alpha(z)} = A + B\rho(z) + \cdots, \tag{15}$$

to determine the constants, A, B, ..., by means of the data listed in Table 1. Besides, this rearrangement of the power series (13) involves the density range of the outer core to be included in the interval of convergence of (15) which is, strictly speaking, an additional condition to be satisfied.

Table 1

Density ρ, pressure P, velocity v_p, cubic expansion α, and specific heat c_P as functions of depth z in the outer core. The suffixes PEM *and* M1 *indicate the reference models presented by* DZIEWONSKI *et al.* (1975) *and by* ULLMANN *and* PAN'KOV (1976), *respectively. The values of α and c_P are adopted from* STACEY (1977)

z km	$\rho_{(PEM)}$ g cm^{-3}	$P_{(PEM)}$ Mbar	$P_{(M1)}$ Mbar	$v_{p(PEM)}$ km s^{-1}	$v_{p(M1)}$ km s^{-1}	α 10^{-6} K^{-1}	c_P J kg^{-1} K^{-1}
2885.3	9.909	1.3540	1.3540	8.002	8.154	15.7	707
2971	10.043	1.4445	1.4446	8.150	8.293	14.9	704
3071	10.195	1.5497	1.5512	8.317	8.450	14.1	700
3171	10.340	1.6540	1.6565	8.477	8.600	13.6	697
3271	10.478	1.7572	1.7603	8.630	8.741	12.8	693
3371	10.611	1.8592	1.8635	8.777	8.877	12.2	690
3471	10.738	1.9598	1.9650	8.917	9.006	11.7	687
3571	10.859	2.0587	2.0645	9.050	9.129	11.3	684
3671	10.974	2.1558	2.1616	9.176	9.246	10.8	682
3771	11.084	2.2509	2.2568	9.295	9.357	10.5	679
3871	11.189	2.3439	2.3498	9.408	9.463	10.1	677
3971	11.288	2.4345	2.4393	9.514	9.562	9.8	675
4071	11.383	2.5227	2.5271	9.614	9.658	9.6	673
4171	11.473	2.6084	2.6118	9.706	9.748	9.3	671
4271	11.558	2.6913	2.6933	9.792	9.833	9.1	670
4371	11.639	2.7713	2.7722	9.871	9.914	8.9	668
4471	11.716	2.8484	2.8485	9.944	9.991	8.7	667
4571	11.789	2.9224	2.9219	10.009	10.063	8.5	665
4671	11.857	2.9933	2.9912	10.068	10.131	8.4	664
4771	11.922	3.0609	3.0584	10.120	10.195	8.3	663
4871	11.984	3.1252	3.1232	10.166	10.257	8.2	662
4971	12.042	3.1861	3.1845	10.204	10.314	8.1	661
5071	12.096	3.2436	3.2423	10.236	10.368	8.0	660
5153.9	12.139	3.2887	3.2887	10.258	10.410	9.7	659

We note that the passage from (13) to (15) can be avoided as follows: The ratio c_P/α as a function on the depth interval of the outer core is stated to be represented by a TAYLOR expansion, such as

$$\frac{c_P(z)}{\alpha(z)} = \frac{c_P(\bar{z})}{\alpha(\bar{z})} + a_1(z - \bar{z}) + a_2(z - \bar{z})^2 + \cdots, \tag{16}$$

where $a_1, a_2, \ldots$ are constants, and $\bar{z}$ is defined by $\rho(\bar{z}) = \bar{\rho}$. Since the function $\rho(z)$ is one-to-one it has an inverse, $z(\rho)$, which, we assume, can be expanded in a Taylor series. Thus

$$z = \bar{z} + b_1(\rho - \bar{\rho}) + b_2(\rho - \bar{\rho})^2 + \cdots, \tag{17}$$

where $b_1, b_2, \ldots$ are constants. From (16) and (17), after suitable rearrangement, equation (15) arises.

In fitting by the power series (15) it is conspicuous that the data give rise to points $(\rho, c_P/\alpha)$, located by a Cartesian coordinate system, which are on a straight line as a good approximation. Hence, empirically, (15) is reduced to

$$\frac{c_P(z)}{\alpha(z)} = A + B\rho(z). \tag{18}$$

By curve fitting, using linear regression techniques, the constants A and B are determined to be

$$A \approx -129.8750\ [\mathrm{km^2\ s^{-2}}], \qquad B \approx 17.5892, \tag{19}$$

presupposing that ρ is measured in $\mathrm{g\ cm^{-3}}$, the correlation coefficient expressed by $r \approx 0.9995$ being satisfactorily close to 1.

Table 2

Approximations of the Grüneisen parameter γ as a function of depth z in the outer core, obtained from: [*col.* 1] *Vashchenko-Zubarev formulation* (9) *by using* PEM (DZIEWONSKI *et al.*, 1975); [*col.* 2] *Vashchenko-Zubarev formulation* (39) *by using* M1 (ULLMANN *and* PAN'KOV, 1976); [*col.* 3] STACEY (1977); [*col.* 4] *semi-empirical law* (20); [*col.* 5] *power law* (21), *with* (22)

z	$\gamma(z)$				
km	1	2	3	4	5
2885.3		1.344	1.419	1.442	1.422
2971		1.341	1.408	1.420	1.411
3071		1.338	1.397	1.399	1.399
3171		1.336	1.386	1.382	1.387
3271	1.512	1.334	1.375	1.368	1.377
3371	1.510	1.332	1.366	1.357	1.367
3471	1.505	1.330	1.357	1.348	1.358
3571	1.497	1.328	1.348	1.340	1.349
3671	1.486	1.326	1.340	1.333	1.341
3771	1.470	1.324	1.332	1.328	1.333
3871	1.451	1.323	1.325	1.323	1.326
3971	1.427	1.322	1.319	1.318	1.319
4071	1.398	1.320	1.312	1.314	1.313
4171	1.362	1.319	1.307	1.310	1.307
4271	1.319	1.318	1.300	1.306	1.301
4371	1.267	1.317	1.296	1.302	1.296
4471	1.206	1.316	1.290	1.298	1.291
4571	1.135	1.315	1.286	1.293	1.287
4671	1.050	1.314	1.282	1.288	1.282
4771	0.952	1.314	1.278	1.283	1.278
4871	0.837	1.313	1.274	1.277	1.275
4971	0.703	1.312	1.270	1.271	1.271
5071	0.547	1.312	1.267	1.264	1.268
5153.9	0.400	1.311	1.265	1.258	1.265

Making use of (18) and (19) we are now able to write (11) in the form

$$\gamma(z) \approx \frac{[v_p(z)]^2}{17.5892\rho(z) - 129.8750}, \tag{20}$$

taking care that v_p must be measured in km s^{-1} and ρ in g cm^{-3}. As mentioned above, the functions $v_p(z)$ and $\rho(z)$ can be drawn from the model by DZIEWONSKI *et al.* (1975). Estimates of $\gamma(z)$ thus obtained are given in Table 2, column 4. It should be noted, however, that these values are not really independent of STACEY's (1977) values given in column 3, because Stacey's values of α and C_p were used.

In the next subsection the power law

$$\gamma = m\rho^n \tag{21}$$

is preferred to the approximation (21) for computational convenience. The constants, m and n, are determined by means of the values given in Table 2 which are presumed to approximately satisfy this type of equation. Since fitting by (21) is reduced to the problem of fitting by the linear equation $\ln \gamma = \ln m + n \ln \rho$ to values of ρ and γ, again linear regression techniques can be applied which yield

$$m \approx 5.2999, \qquad n \approx -0.5738, \tag{22}$$

provided that ρ is measured in g cm^{-3}. In this case the correlation coefficient becomes $r \approx -0.9890$. We emphasize that (21) with (22) is nevertheless up to standard in estimating the (adiabatic) temperature distribution in the outer core. Values of γ obtained from (21) and (22) are given in Table 2. We observe that $\gamma(z)$ estimated by STACEY (1977) runs much closer to $\gamma(z)$ obtained from (21) and (22) than to $\gamma(z)$ from (20). However, the differences are, in any case, less than the uncertainties.

2.3. A simple temperature approximating function for the outer core

Preferring the density to the volume in (1) and integrating with respect to density from ρ^* to ρ, we get

$$T(\rho, S) = T(\rho^*, S) \exp \int_{\rho^*}^{\rho} \frac{\gamma(\rho', S)}{\rho'} d\rho', \tag{23}$$

where ρ^* as a limit of integration shall suitably be chosen. This formula can be used to estimate the adiabatic temperature distribution in the outer core, since γ depends only upon P and not T and is given by the tabulated $\gamma(z)$ without the need to consider entropy. Using the power law (21) in (23), we obtain

$$T_{ad}(z) \approx T_{ad}(z^*) \exp \frac{\gamma(z) - \gamma(z^*)}{n}, \tag{24}$$

where $\gamma(z)$ and n are determined by using (21) and (22).

The approaches (20) and (21) are applicable also to the estimation of the melting

point T_M as a function of depth z on the outer core. To this end we introduce the Lindemann melting law in the differential form

$$\frac{1}{T_M}\frac{dT_M}{dP} = \frac{2}{\kappa_M}(\gamma - \tfrac{1}{3}), \tag{25}$$

which proves to be suitable for materials with central atomic forces in which melting involves no gross changes in coordination (see STACEY and IRVINE, 1977a, 1977b, and WALZER, 1980). By κ_M the bulk modulus along the melting curve is denoted. Eliminating κ_M by substituting $dP = \kappa_M(d\rho/\rho)$ into (25) which is equivalent to neglecting the difference between κ_S and κ_M, we obtain

$$\frac{dT_M(z)}{T_M(z)} = 2[\gamma(z) - \tfrac{1}{3}]\frac{d\rho(z)}{\rho(z)}. \tag{26}$$

Integrating and taking the antilogarithm

$$T_M(z) = T_M(z^*)\left[\frac{\rho(z^*)}{\rho(z)}\right]^{2/3} \exp\left[2\int_{z^*}^{z}\frac{\gamma(z')}{\rho(z')}\frac{d\rho(z')}{dz'}\,dz'\right]. \tag{27}$$

For further development of this formula the approximations (20) and (21) are available. As a result, from (27), performing the integration by using the power law (21), we have

$$T_M(z) \approx T_M(z^*)\left[\frac{\rho(z^*)}{\rho(z)}\right]^{2/3} \exp\frac{[\rho(z)]^n - [\rho(z^*)]^n}{\frac{1}{2}m^{-1}n}, \tag{28}$$

where the values of m and n are taken from (22). We remember that $\rho(z)$ has been given in analytical form by DZIEWONSKI *et al.* (1975). It should be noted that (28) is equivalent to the approximation of $T_M(z)$ presented by STACEY (1977), except that the constants corresponding to m and n, as given in (22), are somewhat different. Incidentally, the combination of (23) and (27) gives rise to the relation

$$T_M(z) \approx T_M(z^*)\left[\frac{T_{ad}(z)}{T_{ad}(z^*)}\right]^2\left[\frac{\rho(z^*)}{\rho(z)}\right]^{2/3} \tag{29}$$

For the reference level z^*, the inner core boundary is used, because it is assumed that the adiabatic temperature there coincides with the melting point, i.e. $T_{ad}(z^*) = T_M(z^*)$.

2.4. *Application of the internal energy approach*

From ULLMANN and PAN'KOV (1976) we take the function

$$X(x, S) \equiv \tfrac{9}{2}\kappa_0(2 - \kappa_1)^{-2}(x^{(2/3)-(1/3)\kappa_1} - 1)^2 \tag{30}$$

multiplied by V_0 to approximate the internal energy U of highly compressed matter such as in the Earth's deep interior. The material parameters κ_0 and $\kappa_1 > 2$, depending on S, are the zero-pressure values of $\kappa_S = k(P, S)$ and $(\partial\kappa_S/\partial P)_S$, respectively. Thus,

applying (30) to the outer core and again assuming entropy changes in this region to be insignificant, we have (with average entropy $\tilde{S}$)

$$U \approx V_0 X(x, \tilde{S}), \qquad P \approx -\frac{dX(x, \tilde{S})}{dx}, \qquad \kappa_S \approx x \frac{d^2 X(x, \tilde{S})}{dx^2} \tag{31}$$

and

$$\left(\frac{\partial \kappa_S}{\partial P}\right)_S \approx -1 - \frac{d}{d \ln x} \ln \frac{d^2 X(x, \tilde{S})}{dx^2}. \tag{32}$$

This system of approximate equations is henceforth called model 1, abbreviated M1. Now it is understood that κ_0 and κ_1 are given by

$$\kappa_0 = k(0, \tilde{S}) \approx \frac{d^2}{dx^2} X(1, \tilde{S}) \tag{33}$$

and

$$\kappa_1 = \frac{d}{dP} k(0, \tilde{S}) \approx -1 - \frac{d}{dx} \ln \frac{d^2}{dx^2} X(1, \tilde{S}). \tag{34}$$

Particularly, as to $P \approx -dX(x, \tilde{S})/dx$, we only indicate that for convenient values of κ_1 this relationship fits into the most important approximations from among those of the equation of state which are customarily said to be of the Gilvarry type (see, e.g., ANDERSON, 1967). The choice of the parameter $\kappa_1 = 4$ leads to the standard form of Birch's equation; if $\kappa_1 = 3$ we obtain the abridged Bardeen equation. In general the equation coincides with a Born-Mie equation having indices in the ratio 2:1. For more details: see ULLMANN and PAN'KOV (1976). We have fitted M1 to the outer-core values of ρ, $v_p = \sqrt{\kappa_S/\rho}$ and P from the parametric Earth model (PEM) presented by DZIEWONSKI *et al.* (1975) to find the parameters

$$\rho_0 \approx 6.7099 \text{ [g cm}^{-3}\text{]}, \qquad \kappa_0 \approx 1.4226 \text{ [Mbar]}, \qquad \kappa_1 \approx 4.5550, \tag{35}$$

recalling that these values are referred to outer-core conditions. In Table 1, values of P and v_p which arise from M1 by using (35) are compared with those of PEM. We observe that the corresponding values both of P and of v_p at each depth z are, on the whole, in quite good agreement.

Let us now use M1 to determine γ through the Vashchenko-Zubarev formula (9) with the relationships (31) and (32)

$$\gamma \approx \left[\frac{2}{9x} \frac{d}{dx} \ln \left|\frac{dX(x, \tilde{S})}{dx}\right| + \frac{1}{2} \frac{d}{d \ln x} \ln \frac{d^2 X(x, \tilde{S})}{dx^2} + \frac{4}{3}\right] \times \left[\frac{4}{3x} \frac{d}{dx} \ln \left|\frac{dX(x, \tilde{S})}{dx}\right| - 1\right]^{-1}. \tag{36}$$

Expanding the function $X(x, \tilde{S})$ according to (30), this becomes

$$\gamma \approx \tfrac{1}{3}(\kappa_1 - 1) + \tfrac{1}{6}(\kappa_1 - 2)(\kappa_1 - 3)[(2\kappa_1 - 5)x^{(2/3)-(1/3)\kappa_1} - \kappa_1 + 3]^{-1}. \tag{37}$$

In Table 2, values of γ calculated by using (37), (35) and $\rho = \rho(z)$ from PEM are itemized.

Introducing (37) into (23) and performing integration, we obtain

$$T_{ad}(z) \approx T_{ad}(z^*) \frac{\tau(\rho(z); \rho_0, \kappa_1)}{\tau(\rho^*; \rho_0, \kappa_1)}, \tag{38}$$

where

$$\tau(\rho; \rho_0, \kappa_1) \equiv x^{-1/3(\kappa_1 - 1)} \left[\frac{2\kappa_1 - 5}{\kappa_1 - 2} - \frac{\kappa_1 - 3}{\kappa_1 - 2} x^{(1/3)(\kappa_1 - 2)} \right]^{1/2}, \tag{39}$$

$x = \rho_0/\rho$. Further, equation (33) becomes, on using (42),

$$T_M(z) \approx T_M(z^*) \left[\frac{\tau(\rho(z); \rho_0, \kappa_1)}{\tau(\rho^*; \rho_0, \kappa_1)} \right]^2 \left[\frac{\rho^*}{\rho(z)} \right]^{2/3}. \tag{40}$$

As before z^* and ρ^* are referred to the inner-core boundary, that is, by PEM, $z^* = 5153.9$ km, $\rho^* = 12.139$ g cm^{-3}.

The application of the internal energy U of the outer-core material at any depth z, approximated by $V_0 X(x, S^*)$ with the parameters (35), to derive an approximate function for the temperature $T_{ad}(z)$ through the Vashchenko-Zubarev formulation of γ, against which physically well-founded objections have been raised, serves to test the fitness of that formulation for a thermal model of the outer core. Since we believe that (24) will help us to obtain reliable information, we therefore make a comparison between (38) and (24). In doing so, attention is confined to the expressions $\tau(\rho(z); \rho_0, \kappa_1)/\tau(\rho^*; \rho_0, \kappa_1)$ and $\exp[\gamma(z)/n - \gamma(z^*)/n]$ with the function $\gamma(z)$ after (21) and (22). The result can be seen in Table 3.

We realize that these two expressions differ from one another only slightly over the whole depth range. The maximum divergence occurs at the core-mantle boundary, the percentage deviation being about 0.3%. From this we come to the conclusion that formula (38) is adequate for our purpose, too. That implies that the Vashchenko-Zubarev approximation of γ in the form (36) in spite of a certain physical insufficiency, is a suitable basis for a thermal model of the outer core as well as the formulas (21) and (20), which bear comparison with the expression for $\gamma(z)$ found by STACEY (1977) as shown in Table 2.

We emphasize that M1 is used as a catalyst only; it merely represents the PEM parameters by continuous analytical functions to a good approximation. This facilitates study of the physico-chemical interpretation of the parameters in (35), which is the subject of a separate paper.

3. *Elimination of the core paradox*

From the preceding section it can be seen that we, unlike HIGGINS and KENNEDY (1971) consider γ to be a function of V. As shown in Table 2, it results in a slight

Table 3

Comparison between the temperature distribution functions

$$u(z, z^*) \equiv \frac{\tau(\rho(z); \rho_0, \kappa_1)}{\tau(\rho(z^*); \rho_0, \kappa_1)}$$

and

$$v(z, z^*) \equiv \exp \frac{[\rho(z)]^n - [\rho(z^*)]^n}{m^{-1}n}$$

(see text) with a view to estimating the fitness of the Vashchenko-Zubarev formulation of the Grüneisen parameter for a thermal model of the outer core

Depth km	$u(z, z^*)$	$v(z, z^*)$
2885.3	0.7639	0.7615
2971	0.7779	0.7762
3071	0.7936	0.7927
3171	0.8087	0.8085
3271	0.8233	0.8235
3371	0.8372	0.8378
3471	0.8505	0.8515
3571	0.8633	0.8645
3671	0.8754	0.8768
3771	0.8871	0.8886
3871	0.8982	0.8998
3971	0.9088	0.9104
4071	0.9189	0.9205
4171	0.9285	0.9300
4271	0.9376	0.9390
4371	0.9463	0.9476
4471	0.9545	0.9557
4571	0.9623	0.9633
4671	0.9696	0.9706
4771	0.9766	0.9773
4871	0.9832	0.9838
4971	0.9895	0.9898
5071	0.9954	0.9955
5153.9	1	1

decrease of γ with rising pressure. In contrast to this, CHAN, SPETZLER, and MEYER (1976), experimentally detected that, in the case of the liquid metals mercury and bismuth, γ slightly increases with increasing pressure. BOEHLER and KENNEDY (1977) obtained the same result for γ of liquid mercury. However, we do not believe these results to be of importance for the Earth's outer core: First, the pressures used in experiments are too small to obtain so close a packing as in the core, and, second, Bi and Hg show anomalous properties. Our theory does not apply to pressures as low as these. It does apply, however, to those in the lower mantle and the core. Let us now turn

to the core paradox referred to in Section 1 by using our theory developed in Section 2. KENNEDY and HIGGINS (1973) themselves offered a solution to that contradiction. They observed that in their investigation of 1971, a layer of about 200 to 300 km thickness at the bottom of the core was overlooked where the adiabatic temperatures for different constant γ-values are just above the melting temperature. We calculated anew the adiabatic temperature according to our method and found the gradient of adiabatic temperature in the outer core *everywhere* to be greater than the gradient of melting temperature according to KENNEDY and HIGGINS (1973). From this it follows either that thermal convection cannot exist anywhere in the outer core (see Fig. 1), or that the interpretation of IOB as the melting point of a material which is chemically uniform both inside and outside, is incorrect. Convection in a layer of 200 to 300 km at the bottom of the outer core can be excluded in any case.

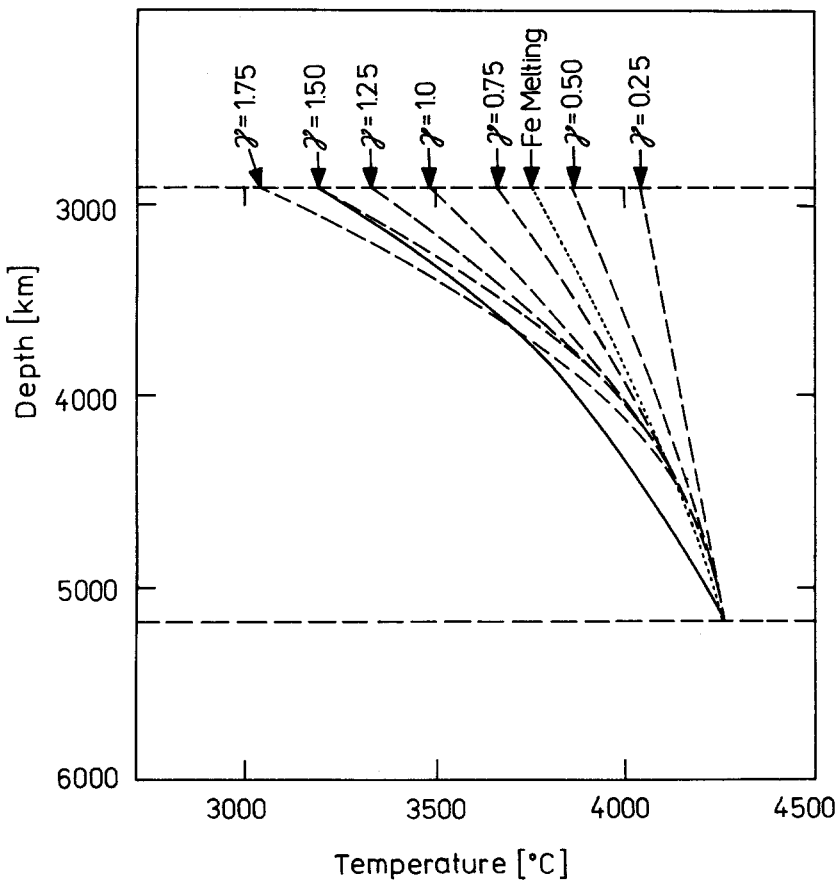

Figure 1
Renewed core paradox for the melting-point curve of iron by KENNEDY and HIGGINS (1973) [= dotted line] in connection with our curve of adiabatic temperature [= full line]. The curves of the adiabatic temperature for constant values of γ according to KENNEDY and HIGGINS (1973) are drawn in dashed lines.

If one considers the linear melting point–pressure relation of KRAUT and KENNEDY (1966) to be the most realistic one, as did CHAN *et al.* (1976), p. 275, although it is not theoretically founded to a sufficient degree, then there is a possibility to circumvent the paradox. The chemical composition of the inner core differs from that of the outer core in that the melting temperature at IOB shows a jump. Melting temperature is significantly higher inside than outside so that an adiabatic temperature curve which is identical with the actual one, lies below the melting temperature curve in the inner core, and above the melting temperature curve in the outer core. Nevertheless, the adiabatic temperature gradient in the outer core may exceed the melting temperature gradient. This realistic suggestion originates in STACEY (1972) (although no longer favoured by him). He specialized it in so far as he assumed pure iron or iron and nickel for the inner core and, as did MURTHY and HALL (1970), LEWIS (1971) and GOETTEL (1972), iron, FeS and small quantities of K^{40} for the outer core, i.e. sufficient K^{40} to generate 10^{13} W. From this it follows that 30% of the heat flux measured on the Earth's surface, originate in the core, and that the geodynamo is driven by thermal convection. This would mean that the lower mantle is heated not only from inside but also from below.

Internal heating of the lower mantle is weak. However, due to the exponential decrease in viscosity with rising temperature, internal heating causes the Rayleigh number of the lower mantle to rise above the critical value from time to time. This leads to episodic lower-mantle convection (WALZER, 1974, 1978). In times of intensified magmatic activities of the Earth (GASTIL, 1960), the lower mantle emits more heat than is normally the case, and lower-mantle convection disappears owing to this and owing to the dependence of viscosity on temperature.

Supposing that additional heat is generated in the outer core by K^{40}, this would be stored by the fusing of a thin layer at the bottom of the lower mantle till another period of lower-mantle convection begins. STACEY (1972) already remarked that, if the actual temperature in the outer core is equal to the adiabatic one, not only heat sources are required in or below the outer core, but also heat sinks at CMB.

It may be stated that this explanation can be realized also when assuming melting-point curves other than that of the Kraut-Kennedy type (see VERHOOGEN, 1973). STACEY's suggestion (1972) p. 111, that the inner and outer cores, from a chemical point of view, differ considerably is based on a density jump between 1 and 2 g/cm^3 at IOB. Modern Earth models, however, show lower jumps: DZIEWONSKI *et al.* (1975) 0.565 g/cm^3 and model B1 of JORDAN (1973) 0.17 g/cm^3. A higher accuracy in the determination of this density jump could bring about a decision on the correctness of Stacey's suggestion since metals during their solidification show but small density jumps.

Criticisms of the melting-point curve used by KENNEDY and HIGGINS (1973) start from different points. Often it is pointed out that their theoretical reasoning was incomplete. This is partially true, but it also applies to the derivation of some other melting-point curves. GILVARRY (1966) pointed out that the Kraut-Kennedy law is consistent with the Lindemann law if γ has a particular strong density dependence. A

serious objection is that the Kennedy-Higgins curve is an extrapolation of experimental melting-point curves for Fe and some other metals from a pressure range between 0 and 40 kbar to pressures up to 3000 kbar (LEPPALUOTO, 1972; VERHOOGEN, 1973). This extrapolation which certainly is inadmissibly extensive, was supported by the observation that the melting-point temperature relationships for potassium, sodium, and rubidium were linear with isothermal volume compressions up to 35%. However, these metals show the strong density dependence of γ required for compatibility with the Lindemann law, which iron does not.

LEPPALUOTO (1972) stated that usual melting-point theories proceeded from solid state physics because the latter is more advanced than the theory of fluids. During melting, however, solid and liquid are in equilibrium so that the other side should also be considered. Leppaluoto did this, using Eyring's significant structure theory of fluids. Equating the Gibbs free energies of liquid and solid, he obtained the melting-point curve for iron shown in Fig. 2 which shows that, in conjunction with our adiabatic temperature curve, thermal convection would be possible everywhere in the outer core. For the lower half of the outer core, the temperature would lie immediately above the melting-point temperature. The relevant activation volume, however, cannot be determined with certainty, so that the quality of Leppaluoto's curve can only be judged with difficulty.

BOSCHI (1975) investigated close-packed structures on model systems of hard spheres. Thus, he calculated the melting temperature of iron by means of a Monte-Carlo procedure. Figure 3 shows that his curve (dashed, 2) with our adiabatic

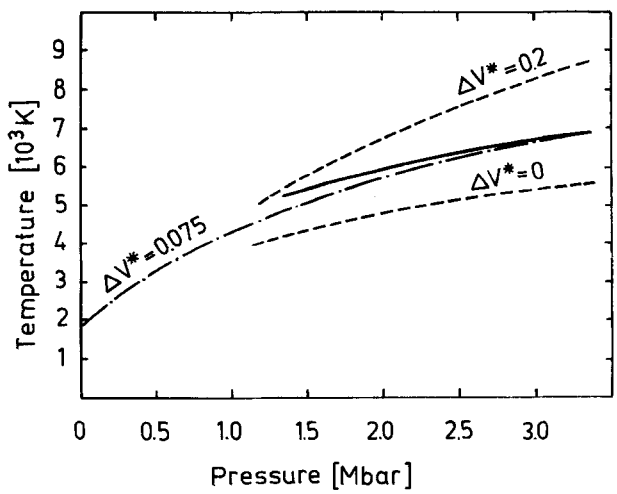

Figure 2

Eliminated core paradox. The melting temperature versus pressure curve for iron according to LEPPALUOTO (1972) for an activation volume $\Delta V^* = 0.075$ in units of cm^3/mole is drawn in a dash-and-dot line. The full line shows our adiabatic temperature for the case that IOB represents a boundary between molten and solid material. A vanishing activation volume marks a curve which essentially resembles that by HIGGINS and KENNEDY (1971).

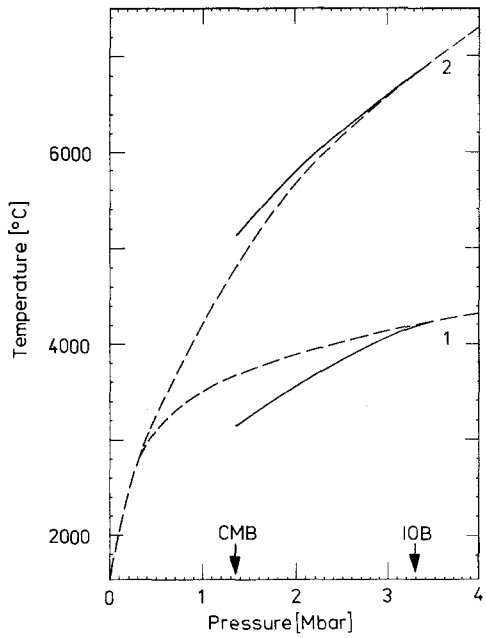

Figure 3

The dashed line 2 shows BOSCHI's (1975) melting temperature curve for iron, the full line 2 our associated adiabatic temperature. The dashed line 1 represents the melting temperature curve for iron by HIGGINS and KENNEDY (1971), the full line 1 our associated adiabatic temperature.

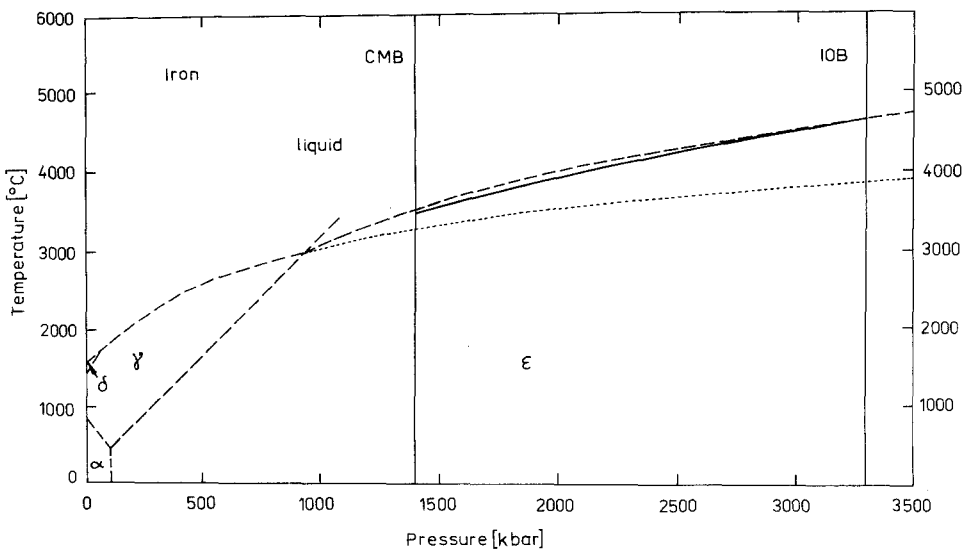

Figure 4

Renewed core paradox. The full line represents our adiabatic temperature curve, the dashed lines the boundaries between the phases of iron α, γ, δ, ε, l (= liquid) according to LIU (1975).

temperature curve 2 permits thermal convection throughout the outer core in case that IOB is a melting boundary. As shown once more by the curves 1 in Fig. 3, this would be forbidden *throughout* the outer core for Higgins' and Kennedy's melting-point curve and our associated adiabatic curve.

The hitherto mentioned melting temperature curves for iron can be criticized in that no phase transitions were considered for them. A suggestion was made, stating that the electronic structure of iron which is approximately $3d^{7.0}\ 4s^{1.0}$ on the Earth's surface, changed towards the state $3d^8\ 4s^0$ at IOB, and that the inner core was glassy. This would obviate the interpretation of IOB as a liquid–solid transition. A quantum-mechanical calculation of the electronic band structure of iron, however, revealed that this transition takes place only with a fourfold compression (BUKOWINSKI, 1976). Thus, it will be realistic, for earthly conditions, to restrict oneself to the well-known phases of iron. LIU (1975) concluded from the extrapolation of experimental data that the triple point of face-centered-cubic (γ), hexagonal-close-packed (ε) and liquid (l) iron is at (0.94 ± 0.20) Mbar and (2970 ± 200)°C. The calculation of the $\varepsilon - l$ boundary, however, is critical. Figure 4 shows that this boundary approaches to our adiabatic curve which we again calculated assuming that IOB corresponds to the liquid–solid transition of a material that is uniform inside and outside. This result is comparable with JACOBS' (1975) expectation that actual temperatures in the outer core are very close

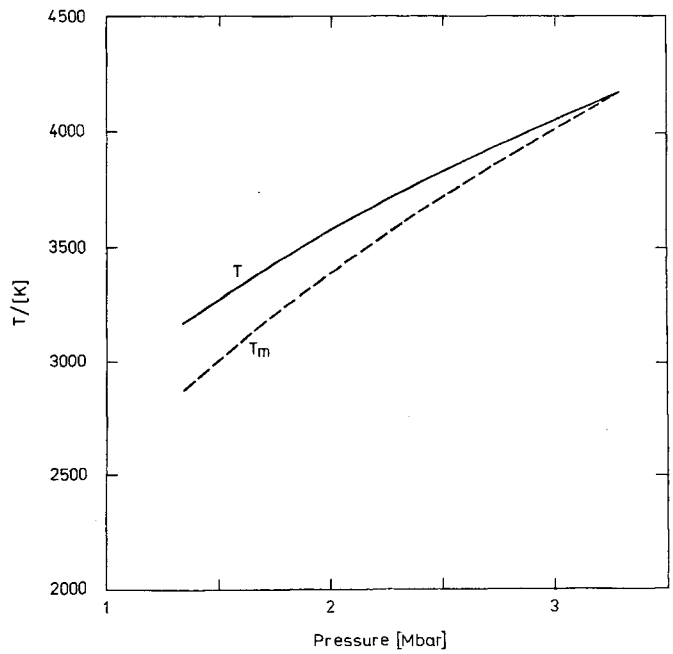

Figure 5
Eliminated core paradox. The full line is our adiabatic temperature curve, the dashed line represents STACEY's (1977) melting temperature curve.

to the melting temperature. In Fig. 4, the full line represents the adiabatic temperature taking into account the electronic contribution to the Grüneisen parameter. Because of the drawing accuracy, it is unimportant whether the semiempirical law (20) or the power law (21) is used for computation of γ. The result shown in Fig. 4 seems remarkable to us; for Liu's melting curve the core paradox exists, too. If, however, the Vashchenko-Zubarev formulation of γ and the equation of state M1 are applied then an adiabatic temperature curve arises which runs very close above Liu's melting-point curve everywhere. The strength of the evidence favouring the Lindemann melting formula (e.g. STACEY and IRVINE, 1977a,b and WALZER, 1980) reinforce the dismissal of the core paradox as non-existent.

4. *Conclusion*

The existence of a core paradox depends upon the assumption of a melting curve with a lower gradient than is indicated by any sound melting theory. This conclusion is not altered by the presence of a light ingredient, FeS (MURTHY and HALL, 1970), FeO (DUBROVSKII and PAN'KOV, 1972) or MgO (ITO, 1976), provided that the melting composition is independent of pressure.

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