THE CORE PARADOX RECONSIDERED *

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Using a new theory of the volume dependence of Grüneisen's parameter we have reconsidered the core paradox. If the melting-point curve according to Higgins and Kennedy (1971) applies, and if the inner and outer cores are chemically equal, thermo-convective dynamos are impossible throughout the outer core. Thermal convection would be prevented even in the 200–300 km thick layer at the bottom of the outer core where, for constant γ-values, thermal convection is possible. Using Leppaluoto's (1972) and Boschi's (1975) melting-point curves and the volume-dependent γ, thermal convection is possible. Liu's (1975) melting-point curve shows the phenomenon of polymorphism of iron. The curve of adiabatic temperature computed by using our theory runs very closely above this, and consideration of the electronic contribution to Grüneisen's parameter produces a curve which runs very close below Liu's curve. By using the other melting-point curves mentioned above the conclusions are not altered. Thus we see that the core paradox exists not only in the case of Higgins' and Kennedy's (1971) melting-point curve and, despite claims to the contrary, this problem is unresolved even today. Various possibilities to circumvent the paradox are discussed.

1. Introduction

We present an investigation of the temperature distribution within the Earth's outer core and of the core paradox. At present, the generation of the geomagnetic main field is chiefly explained by a dynamo mechanism. As is well known, each dynamo must satisfy two fundamental theorems: (1) the magnetic field generated by the dynamo must be neither axisymmetric nor two-dimensional; (2) mass flow connected with the dynamo must not be purely toroidal, i.e., the radial component of the velocity field must not disappear. The second theorem leads directly to the topic of this paper. To obtain a steady-state radial velocity component of flow within the outer core, thermal convection is usually assumed to exist there. Higgins and Kennedy (1971), however, have claimed that the liquid outer core has a stable, stratified structure and thermal convection, therefore, should be impossible. This claim is based on an extrapolation of the melting-point curve of Kraut and Kennedy (1966) which shows a gradient with depth smaller than that of the adiabatic temperature curve. If we assume the outer and inner cores to be of the same chemical composition, and the temperature at the boundary between the liquid outer core and the solid inner core to correspond to the melting point of the material at the pressure existing there, it follows that the adiabatic temperature everywhere in the outer core is below the melting temperature. However, since the outer core is molten, the temperature cannot coincide with adiabatic temperature, which should be the case in practice if thermal convection were to exist. Consequently, thermal convection is not permissible, which contradicts the requirement of dynamo theory. In the following we will reconsider this so-called "core paradox". The core paradox arises if the gradient of the adiabatic temperature is greater than the gradient of the melting temperature in the whole outer core. It is evident that the melting-point curve does not

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necessarily have to be that of Higgins and Kennedy (1971). Other possible difficulties of Higgins' and Kennedy's (1971) paradox have been discussed by Verhoogen (1973) and Frazer (1973).

2. Models

It is well known that Kennedy and Higgins (1973) have offered a solution to the core paradox. Reconsidering the problem they found that in their earlier investigation a layer of ~200–300 km thickness at the bottom of the outer core had been overlooked where the adiabatic temperature for different constant γ-values is just above the melting temperature. We, however, took into consideration the volume dependence of Grüneisen's ratio γ. For this purpose we used the formula of Vashchenko and Zubarev (1963) (see eq. 4).

It was shown by Ullmann and Pan'kov (1976) that the approximation function

$$X(x, \xi) = (9/2)\,[\kappa_0/(2-\kappa_1)^2]\,[x^{(2/3-1/3\kappa_1)} - 1]^2 \quad (1)$$

for the strain energy of a hydrostatically compressed body, measured per unit volume of the undeformed body, can, in particular, be used to advantage for the elucidation of physical properties of the Earth's deep interior. The volumetric contraction is defined by the ratio of densities ρ_0 and ρ of the body in its uncompressed and compressed state, respectively, i.e., $x \equiv \rho_0/\rho$. The symbol ξ represents the temperature T or, alternatively, the entropy S. The material parameters κ_0 and $\kappa_1 > 2$, depending on ξ, are the incompressibility and its pressure derivative, respectively, both quantities relating to the zero-pressure state of the body. Hence, for the pressure P, the incompressibility $\kappa = k(P, \xi)$ and its pressure derivative, $(\partial/\partial P)k(P, \xi)$, we may write, approximately

$$P = -\partial X(x, \xi)/\partial x\,, \quad \kappa = x\partial^2 X(x, \xi)/\partial x^2$$

$$(\partial\kappa/\partial P)_\xi = -1 - [(x^2/\kappa)(d^3 X(x, \xi)/\partial x^3)] \quad (2)$$

We call the above equations Model 1 (M1). Walzer et al. (1979) compared three equation-of-state theories by using experimental high-compression data on 17 cubic substances and found good agreement with the measured values for M1. Assuming that the matter in the Earth's outer core can be represented by a homogeneous monophase system, and that the consideration of deviations from adiabaticity within this depth range is not necessary on the basis of the available observational evidence, we calculated the corresponding values of the material parameters ρ_0, κ_0 and κ_1 by fitting M1 to the values of ρ, P and κ in the outer core which result from the parameterized Earth models (PEM) developed by Dziewonski et al. (1975). We thus obtained

$$\rho_0 \approx 6.710 \text{ g cm}^{-3}, \quad \kappa_0 \approx 1.423 \text{ Mbar}, \quad \kappa_1 \approx 4.555 \quad (3)$$

provided that these initial values are defined under adiabatic conditions ($\xi \equiv S$ = constant). In Table 1, values of pressure (P) and of velocity (v_p) of seismic compressional waves resulting from eqs. 1–3 are compared with those computed by using PEM. Since the outer-core matter reacts to seismic wave propagation as a fluid medium, we may put $[k(P, S)]^{1/2} = v_p$. It is noticed that the values P_{PEM} and P_{M1}, as well as v_{pPEM} and v_{pM1}, are in good agreement.

Irvine and Stacey (1975) derived a relationship for the Grüneisen parameter γ taking the form

$$\gamma = \left(\frac{1}{2}\frac{\partial\kappa}{\partial P} - \frac{5}{6} + \frac{2}{9}\frac{P}{\kappa}\right)\Big/\left(1 - \frac{4}{3}\frac{P}{\kappa}\right) \quad (4)$$

which was originally determined by Vashchenko and Zubarev (1963) from quite different considerations. This formula was derived presuming three-dimensional thermal oscillations of atoms in a close-packed

TABLE I

Pressure P, compressional velocity v_p and Grüneisen's parameter γ as functions of depth d in the Earth's outer core. The suffixes PEM and M1 indicate the reference models used by Dziewonski et al. (1975) and Ullmann and Pan'kov (1976), respectively

d (km)	P_{PEM} (Mbar)	P_{M1} (Mbar)	$v_{p,PEM}$ (km s^{-1})	$v_{p,M1}$ (km s^{-1})	γ
2885.3	1.3540	1.3540	8.002	8.154	1.3437
3071.0	1.5497	1.5512	8.317	8.450	1.3385
3371.0	1.8592	1.8635	8.777	8.877	1.3316
3671.0	2.1558	2.1616	9.176	9.246	1.3261
3971.0	2.4345	2.4393	9.514	9.562	1.3216
4271.0	2.6913	2.6933	9.792	9.833	1.3181
4571.0	2.9224	2.9219	10.009	10.063	1.3152
4871.0	3.1252	3.1232	10.166	10.257	1.3128
5153.9	3.2887	3.2887	10.258	10.410	1.3110

atomic arrangement. We think that fluids subjected to pressures between 1.35 and 3.29 Mbar must contain many clusters of densely packed atoms (or molecules).

The application of eq. 4 to the fluid outer core can be justified by two facts. At pressures around 2.420 Mbar (see Table I) the matter must approach a close-packed atomic arrangement and, the frequency of the thermal atomic oscillations is much greater than the frequency of diffusive transitions, so that the atoms develop a structure even though this may be transitional. Moreover, in eq. 4 we emphasize that P and κ are corresponding zero-temperature extrapolations. Since, conversely, from various considerations it follows that γ is virtually independent of temperature (T), even under outer-core conditions, we are justified in replacing eq. 4 by the approximation

$$\gamma = \frac{\kappa_1 - 1}{3} + (1/6)\left[\frac{(\kappa_1 - 2)(\kappa_1 - 3)}{(2\kappa_1 - 5)x^{2/3-(1/3)\kappa_1} - \kappa_1 + 3}\right] \quad (5)$$

which arises from substituting eqs. 1 and 2 in eq. 4. In Table I numerical values of γ are itemized, calculated from eq. 5 by using the estimates of ρ_0 and κ_1 noted in eq. 3. For the mean of γ, the variation of which is only slight in the outer core, we thus obtained a value of 1.3233. This value is supported by the results of other workers, who have obtained values of about 1.4 (Irvine and Stacey, 1975) and 1.5 (Anderson, 1979).

From thermodynamic relationships it follows that

$$(\partial \ln T/\partial \ln x)_S = -\gamma \quad (6)$$

This, together with eq. 5, appears to give reliable information on the temperature field in the depth range of the outer core over which the temperature gradient is widely believed to be adiabatic. Equation 6 may be rewritten as

$$T = T_* \exp\left[\int_x^{x_*} \gamma(u)\,\frac{\mathrm{d}u}{u}\right] \quad (7)$$

provided that $S = S_* =$ constant. Temperature (T_*), entropy (S_*) and volume contraction (x_*) relate to any fixed place within the outer core or at one of its two boundaries. Hence, incorporating eq. 5, after integration we have

$$T = T_*[\tau(x;\kappa_1)/\tau(x_*;\kappa_1)] \quad (8)$$

where

$$\tau(x;\kappa_1) \equiv x^{-(1/3)(\kappa_1-1)} \times\left[\frac{2\kappa_1 - 5 - (\kappa_1 - 3)x^{(1/3)(\kappa_1-2)}}{\kappa_1 - 2}\right]^{1/2}$$

$$\tau(1,\kappa_1) = 1 \quad (9)$$

For the outer core we assume, according to PEM, the lower and the upper limits of density (ρ), which is expected to be a monotonous, smooth function of depth (d) are 9.909 and 12.139 g cm^{-3}, respectively. Consequently, recalling eq. 3 and putting $\tau(6.710/\rho;\ 4.555) \equiv \tau_{OC}(\rho)$, ρ expressed in g cm^{-3}, for the depth range $2885.3 \leqslant d \leqslant 5153.9$ km (see Table I) eq. 9 can be modified to

$$\tau_{OC}(\rho) = 0.1\,[1.7666\rho^{2.37} - 3.3815\rho^{1.5183})^{1/2}$$

$$9.909 \leqslant \rho \leqslant 12.139 \text{ g cm}^{-3} \quad (10)$$

We now refer the values T_* and x_* in eq. 8 to the inner-core–outer-core boundary (IOB). Hence $\rho_* =$ 12.139 g cm^{-3}. Since, in view of eqs. 8 and 10, $T/T_* = \tau_{OC}(\rho)/\tau_{OC}(\rho_*)$ is valid, eq. 8, being the basic relation for our subsequent discussion, takes its final form

$$T = (T_{IOB}/1000)(349.1957\rho^{2.37} - 668.4082\rho^{1.5182})^{1/2}$$

$$9.909 \leqslant \rho \leqslant 12.139 \text{ g cm}^{-3} \quad (11)$$

where $T_{IOB} \equiv T_*$ denotes a reliable estimate of the adiabatic temperature at IOB. From this, in particular, it is easily seen that the temperature T appears to reach a value at IOB which is ~31% higher than the one at the core–mantle boundary (CMB).

3. Discussion

Using the volume dependent γ we recalculated the adiabatic temperature according to our method and found the gradient of adiabatic temperature in the outer core to be everywhere greater than the gradient of melting temperature according to Kennedy and Higgins (1973). From Fig. 1 it follows either that thermal convection cannot exist anywhere in the outer core, or that the interpretation of IOB as the melting point of a material which is uniform both inside and outside, is incorrect. Convection in a layer of 200–

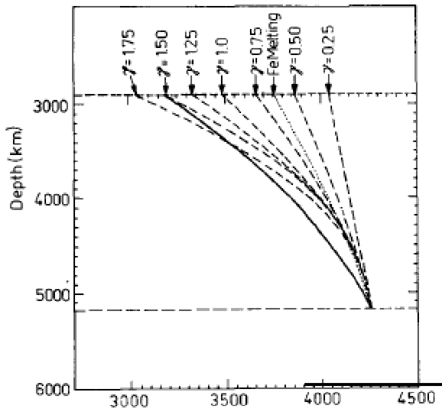

Fig. 1. The solid line denotes our adiabatic-temperature curve using the volume dependence of γ. The curves of the adiabatic temperature for several constant γ-values are shown as broken lines, the melting-point curve of iron is a dotted line. The latter curves are taken from Kennedy and Higgins (1973).

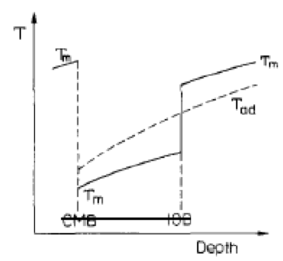

Fig. 2. Pictorial representation of the proposal of Stacey (1972) circumventing the core paradox: inner and outer cores differ in their chemical composition so that a jump of the melting-point versus depth curve occurs.

300 km at the bottom of the outer core can be excluded in any case.

If one considers the melting point–pressure curve of Higgins and Kennedy (1971) to be the most realistic one, then there are the following possibilities to circumvent the paradox:

(1) Thermal convection and precession as causes of the geomagnetic field are excluded. Thus, the necessity to construct a dynamo for a stably-stratified outer core, or to find another kind of magnetic field generation, would arise. One approach to obtain a radial component of stream velocity under these circumstances is gravitational differentiation in the outer core (Loper and Roberts, 1978).

(2) After Jacobs (1976), Kennedy recently considered the Grüneisen ratio for liquid iron in the outer core to be about 0.1. Thus, adiabatic temperature would be higher than melting temperature. In this case, Jacobs' (1976) theory of the generation of the fluid outer core, as well as the theory of the thermoconvective dynamo could be retained. However, a value as extremely low as 0.1 is in contradiction to all well known estimates of Grüneisen's ratio for the outer core. Therefore, we believe that we can exclude this possibility.

(3) The chemical composition of the inner core differs from that of the outer core where the melting temperature at IOB undergoes a jump (see Fig. 2). Melting temperature is significantly higher inside than outside, hence an adiabatic temperature curve which is identical to the actual one lies below the melting temperature curve in the inner core and above the melting temperature curve in the outer core. Nevertheless, the adiabatic temperature gradient in the outer core may exceed the melting temperature gradient. This realistic suggestion was originally made by Stacey (1972).

Leppaluoto (1973) has stated that the usual melting-point theories proceeded from solid-state physics because this is a more advanced science than the theory of fluids. During melting, however, solid and liquid are in equilibrium so that both aspects should be considered as Leppaluoto (1972) has done. Equating the free energies of liquid and solid, he determined a melting-point curve for iron. Figure 3 shows that, in conjunction with our adiabatic temperature curve, thermal convection would be possible everywhere in the outer core. In the lower half of the outer core the temperature would lie immediately above the melting-point temperature. The activation volume, however, cannot be determined with certainty, so that the quality of Leppaluoto's curve can only be judged with difficulty.

Boschi (1975) investigated close-packed structures on model systems of hard spheres, calculating the melting temperature of iron by means of a Monte-Carlo prodedure. Figure 4 (broken line 2) shows that his curve, in conjunction with our adiabatic temper-

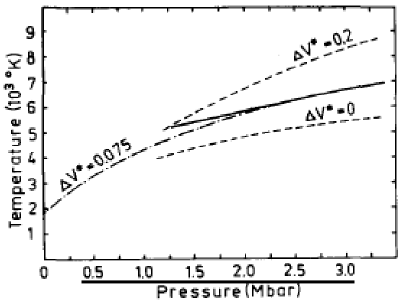

Fig. 3. The broken and dotted line represents the melting-temperature curve by Leppaluoto (1972) for an activation volume $\Delta V^* = 0.075$ cm^3 mol^{-1}. The solid line shows our adiabatic-temperature curve.

ature curve, permits thermal convection throughout the outer core where IOB is a melting boundary. Figure 4 (curves 1) also shows that this process would be forbidden throughout the outer core for Higgins'

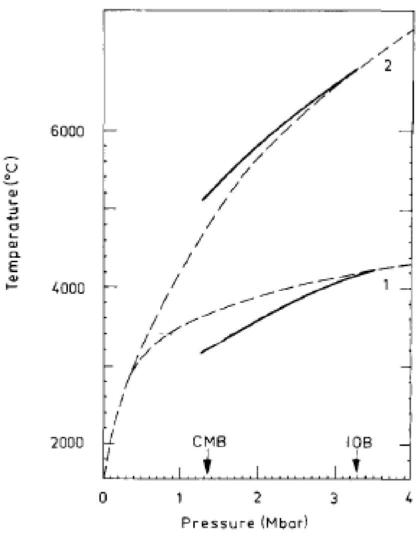

Fig. 4. The solid line (2) is our adiabatic-temperature curve which was computed for the case that IOB is a melting boundary and that the melting-temperature curve of iron (broken line 2) by Boschi (1975) applies. The broken line (1) is the melting-temperature curve of iron by Higgins and Kennedy (1971), the solid line (1) is our appropriate adiabatic temperature.

and Kennedy's (1971) melting-point curve and our appropriate adiabatic curve.

The polymorphism of iron and, especially, the influence of the γ–ϵ transformation on the phase diagram has been thoroughly discussed by Birch (1972) and Liu (1975). In contrast to the three melting curves mentioned above, the influence of the various phases of iron has been considered by Liu (1975). It is realistic, for Earth conditions, to restrict the discussion to the well known phases of iron, i.e., to disregard speculations on a distinct minimum of the melting-point curve of iron (McLachlan and Ehlers, 1971) the more so as Bukowinski (1976) has proved that the change of the electronic structure to a 3 d^8 state occurs at a compression of approximately four times that at the inner-core boundary. Liu (1975) has concluded, from the extrapolation of experimental data, that the triple point of face-centered cubic (γ), hexagonal close-packed (ϵ) and liquid iron is at 0.94 ± 0.20 Mbar and 2970 ± 200°C (see Fig. 5). The calculation of the epsilon-liquid boundary, however, is critical. Figure 5 shows that this boundary approaches our adiabatic curve which we again calculated assuming that IOB corresponds to the liquid–solid transition of a material that is uniform inside and outside. Thus, as in the case of Liu's (1975) melting-point curve, thermal convection would be possible.

A second method for estimating the volume dependence of Grüneisen's ratio γ has been developed involving the electronic contribution to γ. In spite of other theoretical premises, the new curves of adiabatic temperature as functions of pressure differ only slightly from those of Figs. 1, 3, 4 and 5 which have been computed by using M1 and the Vashchenko–Zubarev (1963) formulation. Hence the conclusions on the possibility of thermal convection in the outer core alter in one case only, namely, in that of Liu's (1975) melting-temperature curve, in the derivation of which the known phase transitions of iron have been considered. Figure 6 demonstrates the impossibility of thermal outer-core convection, presupposing the validity of this curve and the recent theory of volume dependence of γ mentioned above.

4. Conclusions

We believe we have shown that, mainly owing to the existing uncertainty regarding melting-point

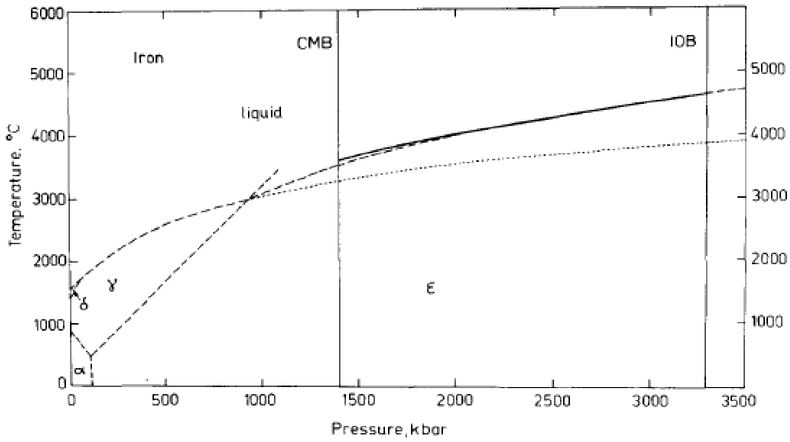

Fig. 5. Broken lines represent the phase boundaries between α, γ, δ, ϵ and liquid iron after Liu (1975). The solid line is the appropriate adiabatic temperature computed by the method discussed in this paper.

curves, the last word on the core paradox has not yet been spoken. It is to be expected that the 10–20% of other materials added to the main constituent iron reduce the absolute value of the melting point by some 100°C, and lower the gradient of the melting-point curve. This is true, independent of whether FeS, FeO or MgO is considered as the light-weight component. Boschi's (1975) melting-point curve of iron is an extreme upper bound. By slightly lowering the melting temperature and its gradient in Boschi's (1975) curve we obtain melting curves of the mixture which resemble those of Leppaluoto (1972), Liu (1975) or Higgins

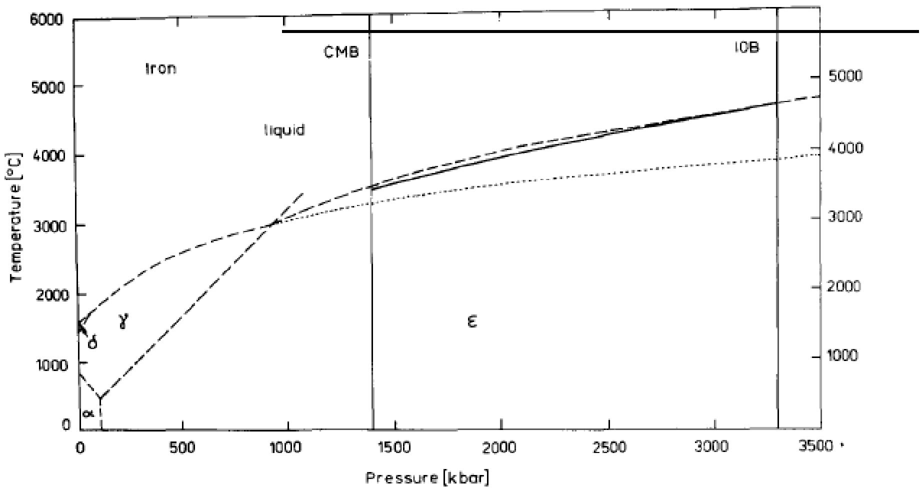

Fig. 6. Broken lines represent the phase boundaries between α, γ, δ, ϵ and liquid iron according to Liu (1975). The solid line is the appropriate adiabatic temperature taking into consideration the electronic part of the Grüneisen ratio.

and Kennedy (1971), depending on the mixture ratio assumed and on the additional compounds. From this it follows that even if we knew the exact law of pressure dependence of the melting temperature it would be impossible to compute the actual effective melting-point curve because of our ignorance of the chemical composition of the admixture with the main constituent iron. That is the reason for the uncertainty over whether the outer core is stably stratified or not. A better melting theory and deeper insight into the chemical composition of the outer core are necessary. One conclusion, however, can be drawn with certainty: if the melting-point curve of Higgins and Kennedy (1971) and our theory of γ apply, and if the inner and outer cores are chemically equal, then thermal convection (in contrast to Kennedy's and Higgins' paper of 1973) is forbidden throughout the outer core, even in the layer near the IOB.

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