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On Intermittent Lower-Mantle Convection

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Abstract

Starting from geochemical results, we assume that the lower mantle has a lower amount of radioactive heat produced per unit volume per unit time than the upper mantle. Consequently, the thermoconvective driving energy in the lower mantle is too weak to permanently maintain convection there. At the same time, however, the lattice and radiative heat conductivities are low there, so that the heat generated is dissipated only to a minor extent. Therefore, temperature increases. As a result, due to the well-known temperature dependence of the effective viscosity, the Rayleigh number increases until the critical Rayleigh number is surpassed and convection starts in the lower mantle, too. Through the currents, heat is dissipated, causing an increase in the permanently existing upper-mantle convection, in magmatism and orogeny as well as earth-wide transgressions. As a result of the heat dissipation, the Rayleigh number again drops beyond the critical value, i.e., due to the low internal heat source density, lower-mantle convection dies down until the next convection interval is started after a long period of heat accumulation. The differential equations of the problem have been reduced to a system of equations with a Hammerstein integral equation and solved numerically. Four convection episodes resulted which agree, in respect of time, with the four highest maxima of Gastil's curve of magmatic activity: These four overturns are found 2820, 3633,4128 and 4496 Ma. after the accretion of the Earth, an age of the Earth of 4600 Ma. being assumed. A comparison of empirical curves showed that these times also correspond to earth-wide transgressions and that the latter are found to lie precisely in the periods in the Phanerozoic in which the geomagnetic dipole field only rarely reversed polarity. The latter most probably has to do with the fact that the lower mantle determines part of the boundary conditions of the hydromagnetic convection in the Earth's outer core.

INTRODUCTION

Convection is nowadays regarded as the decisive motor of geotectonic processes. This conception is well-founded, although a number of other causes probably have a modifying effect. However, from the energetical point of view, these other mechanisms are of minor importance. The only case where this statement cannot be made is the expansion hypothesis, since up to now no verified physical theory exists for it.

The original argument that gave rise to the introduction of the convection theory (Ampferer, 1906) was the observation that the sediments, present nowadays in an imbricated or folded form and shifted upon one another as nappes, would, if straightened out, cover a far greater area than the pertinent crystalline basement. The reason why modern convection theory has been introduced is the observed movement of the lithospheric plates. A third argument: According to Davies (1980b), the average heat flux density of the Earth is (80 ± 8) mW/m^2, that of the continents (55 ± 5) mW/m^2 and that of the oceans (95 ± 10) mW/m^2. But the radioactive heat generation of a 6 km thick oceanic crust contributes only about 0.5 mW/m^2 to the oceanic heat flux density. It can be shown that lattice and radiative heat conductivities of the mantle below it do not suffice to account for the oceanic heat flux observed. Therefore, there must be convection. Further evidence has been given by Walzer (1973). Reference should also be made to the outstanding papers by Schubert (1979) and Turcotte (1979).

On the other hand, the hypothesis of the expanding Earth is a geotectonic hypothesis that cannot be easily refuted. Carey (1976), in an impressive paper, presented an excellent treatise of geological and geophysical observations that can be interpreted in this sense. Taking a purely empirical approach, he draws the conclusion that the Earth expanded more and more rapidly with the result that the Earth's radius had its maximum growth in the Phanerozoic. Schmidt and Embleton (1981), too, assume that there was a strong expansion. They conclude from common apparent polar-wander paths that the Earth grew from a half radius to the present one in the period between 1.6×10^9 years and 1.0×10^9 years.

It might be that the contradiction between the empirical conclusions can be avoided. In any case, only little can be said in physical terms about geological hypotheses of this type, because the mechanisms suggested for them (by other authors) are based on new physical laws still unproven. Wesson (1978) gives a survey on cosmologies with variable gravitational constant. A new hypothesis of this kind was suggested by Grabińska and Zabierowski (1980). It is our view that convection is the most important tectonophysical mechanism, with thermal convection prevailing at least in the Proterozoic and Phanerozoic. The convection models are based on well-known physical laws, the parameters used can also be properly

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approximated, at least their orders of magnitude. However, there are some observations, such as the progressive retreat of the epicontinental seas in the Phanerozoic which, through convection alone, cannot be explained, but quite well through expansion. But it must be underlined that this remark is just a speculation, in contrast to the analytically and numerically computed convection mechanisms that, in part, could be clearly verified (e.g. Walzer, 1973; and this work). We think that, if Earth expansion exists, it must be a background process.

One question that is still partly unsettled within the convection theories is whether, and to what extent, the lower mantle participates in the currents. Up to the middle of the seventies, the view prevailed that convection is confined to the upper mantle. Today, a number of geophysicists (e.g. Sharpe and Peltier, 1979; Elsasser *et al.*, 1979) assume convection cells which reach from the top to the bottom of the mantle. Ringwood (1975, p.520) pointed out that rather high lateral temperature differences would be necessary in this case between the upward current and the downward current, which would result in so high regional gravity differences as are nowhere observed.

A mediating line is taken in the theories assuming separate convection cells in the upper and lower mantles (e.g. Walzer, 1973, 1974; McKenzie and Weiss, 1975; Richter, 1979; Christensen, 1981), which naturally act on one another both thermally and mechanically at the contact face. According to Sammis (1976), a chemically-induced density difference of 0.1% would be sufficient for separating the convection cells. A chemical density jump appears to be plausible for geochemical reasons: Ringwood (1971, 1975) and Dickinson and Luth (1971) suggested that the lower mantle is the heavy remainder of the differentiation of the continental crust. According to Ringwood (1975), the continents contain 30-60% of the uranium and barium of a chondritic primeval Earth, while the upper mantle has the original composition. O'Nions *et al.* (1979) and Jacobsen and Wasserburg (1979) also conclude that the mantle is chemically stratified.

A further important topic is the behaviour of the effective viscosity as a function of depth. As has been shown by Walzer (1978a), viscosity in the lower mantle should be greater than 1.54×10^{26} poise *and* independent of temperature to prevent convection there for all times. At present, there is no consensus on the actual value of the effective lower-mantle viscosity. MacDonald (1963) concluded from the phase delay of the equilibrium figure that the viscosity of the lower mantle is about 10^{26} poise. McKenzie (1967), examining the slow deformations of the Earth and using considerations from solid-state physics, arrived at the conclusion that the viscosity of the lower mantle must be $\sim 10^5$ times greater than that of the upper mantle. The result obtained by Isacks and Molnar (1971), stating that for deep-focus earthquakes (depth >300 km) the compression stresses are parallel to the descending lithospheric slab, also led to the assumption that the slab in this depth range encounters increasing resistance due to growing viscosity. Richter (1977), too, inferred that the stress distribution in the descending lithospheric slab should be attributed to growing viscosity. Schubert and Young (1976) computed that the viscosity of the lower mantle must not be less than 10^{24} poise as, otherwise, the core would have to be solid at the core-mantle boundary. These results have not remained undisputed. Examining the isostatic uplift of regions of different sizes (Laurentia, Fennoscandia, Lake Bonneville) after the melting of the ice, Cathles (1975) and Peltier and Andrews (1976) arrived at very similar results by means of various mathematical methods. For the lower mantle, Cathles finds $(0.9 \pm 0.2) \times 10^{22}$ poise. The entire mantle is to be considered as a Newtonian fluid. Cathles' results for the upper mantle do not basically differ from those of classical and modern authors: Haskell (1935) found from the post-glacial uplift of Fennoscandia 3.10^{21} poise for an underlying viscous half-space, Vetter and Meissner (1977, fig. 10) determined for a depth of 150 km values ranging from 10^{17} to 10^{22} poise, depending on whether the region lies under an ocean or consolidated continent.

What can material physics contribute to the subject considered here? In Figure 1, the experimental results of a great number of authors have been summarized by Ashby and Verrall (1977). Since a growing hydrostatic pressure results in a growing tendency towards the formation of perfect crystals, i.e. the number of lattice defects (dislocations, holes) is reduced, because this facilitates a better utilization of the space, it is to be expected that the (effective) viscosity must increase with depth, if the material remains the same. This applies, in particular, to the transition to denser lattices (phase transition). Therefore, in our view, the assumption of a nearly constant viscosity across almost the entire mantle (Cathles, 1975) is unlikely. No matter whether dislocation glide, dislocation climb or Nabarro-Herring creep prevails, the temperature- and pressure-dependence is defined by

$$D = D_0 \exp(-Q_0/k_0T) \exp(-pV_0/k_0T)$$

and

$$(Q_0 + pV_0)/k_0 = k_2T_m$$

(Weertman and Weertman, 1975), with only the constants being different. It follows from formula

$$\eta = k_1 \exp(k_2T_m/T)$$

that the viscosity even without phase transitions is constant only if T/T_m does not change.

Weertman (1978) now suggests a probably

promising solution to the problem of actual lower-mantle viscosity: Deformations that are due to post-glacial epirogeny are between 10^{-5} and 10^{-6}. But for creep tests on rocks and convection in the mantle, much higher deformations and steady-state (or quasi-stationary) creep must be assumed. Thus, the post-glacial uplift is to be considered as transient creep caused by other micromechanisms, while convection would have to be described by a power law. He (and so do we) concludes that the effective viscosity for the various depths of the mantle varies by four to six orders of magnitude, if a power law applies to convection. (But if one just wants to roughly examine the hydrodynamics of convection, a Newtonian fluid suffices as a first approximation.)

PRELIMINARY GEOPHYSICAL CONSIDERATIONS ON THE COMPUTATION OF THE MECHANISM OF INTERMITTENT CONVECTION IN THE LOWER MANTLE

If one rejects Weertman's (1978) suggestion that transient creep is the decisive factor for the post-glacial uplift, and if one maintains the assumption that a Newtonian fluid plays a part in this process, one arrives at the conclusion (improbable from the point of view of solid-state physics) that the entire mantle has a constant viscosity of about 10^{22} poise (Cathles, 1975; Peltier and Andrews, 1976). This is the reason why, recently, a number of authors have favoured continuous mantle convection with cells reaching from the core-mantle boundary up to the surface of the

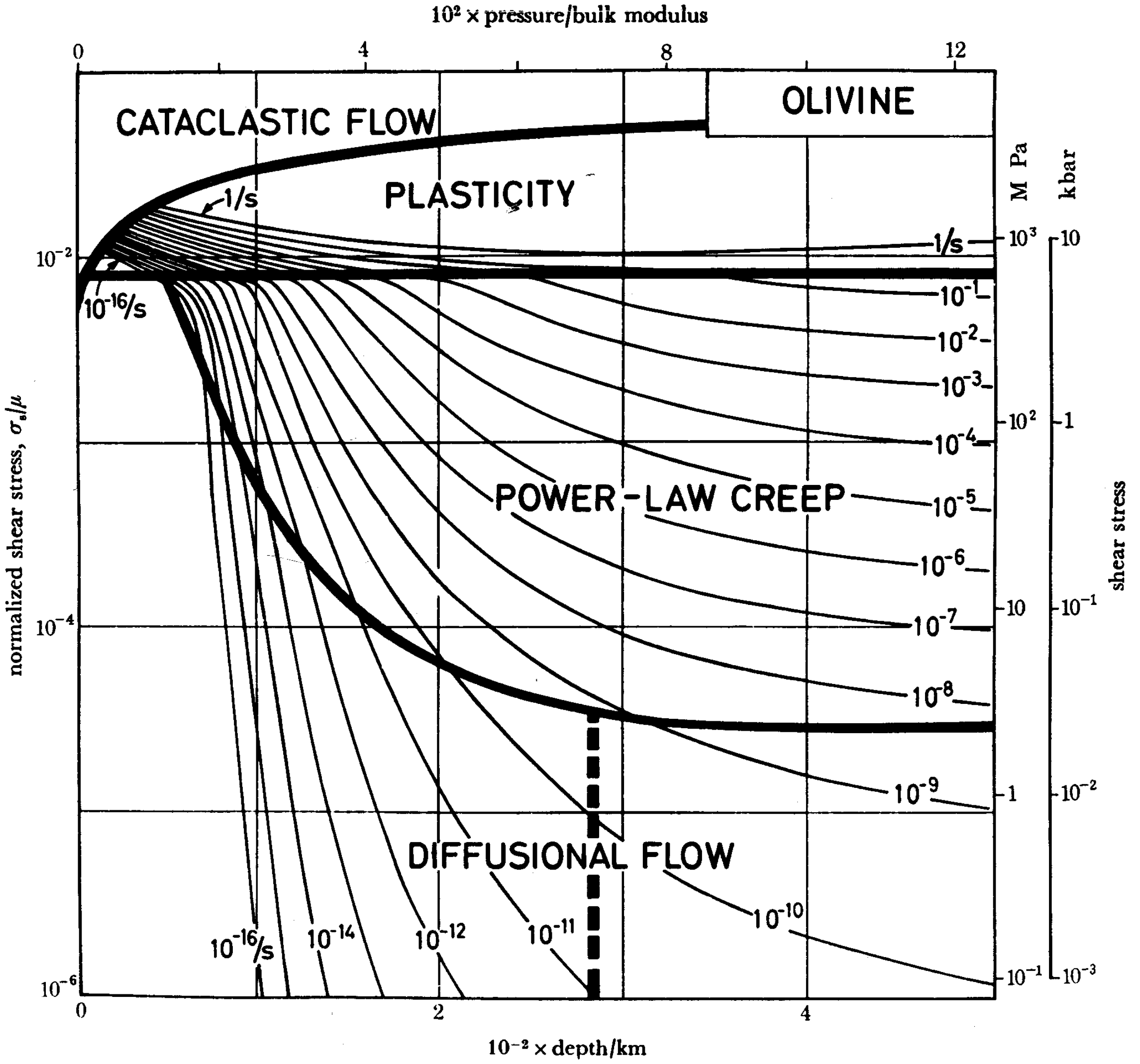

FIGURE 1

The constitutive equations valid after Ashby & Verrall (1977) for olivine under the conditions of the upper mantle. Within *one* area, spreading between shear stress and pressure per bulk modulus, the same constitutive equation holds.

oceanic lithosphere (Davies, 1977; O'Connell, 1977; Schubert, 1979; Elsasser *et al.*, 1979). This view is incompatible with the observation that earthquakes at descending lithospheric slabs reaching only to small up to medium depths have down-dip extension mechanisms, while those reaching down to depths of 600-700 km show compression mechanisms over the entire length of the slab. The latter fact has now been interpreted by the proponents of a mantle of constant viscosity to the effect that the slab dissolves there, while the individual parts continue to sink to a greater depth. In such a case, however, the question arises why seismicity stops to abruptly in the downward direction for, according to this hypothesis, the slab disintegrates with increasing depth so that the earthquakes should become ever rarer and weaker towards the interior. As a matter of fact, however, seismicity shows a distinct maximum between 500 and 700 km and suddenly disappears beyond a depth of 700 km (e.g. Billington, 1978; Hanuš and Vaněk, 1979). It seems that the change in the character of the focal mechanism can only be explained by the assumption that the descending lithospheric slab sinks down to medium depths additionally due to its own weight and, thus, is pulled, while it encounters resistance in greater depths. This resistance might be due to an increase in viscosity (in particular, at the 670 km discontinuity), but also due to the buoyancy in a denser medium in the case of a chemical stratification of the mantle (Richter, 1979).

Attempts to prove the existence of slab extensions beyond a depth of 700 km are based on investigations of S-velocity gradients (Jordan, 1975), but they yield ambiguous results. Davies (1980a), too, discussed the problem of the focal mechanisms of the descending lithospheric slab and has recently also been considering an increase in viscosity with depth. McKenzie and Jarvis (1980), using a thermodynamic approach, found that the order of magnitude of the stress release in an earthquake would be best compatible with a convection layer confined (at present) to the upper mantle. Jeanloz and Richter (1979) conclude from seismic data and from a computation of temperature as a function of depth that the D'' layer above the core-mantle boundary differs chemically from the remaining lower mantle or that the lower and upper mantles have different chemical compositions, with no convection existing (at present) in the lower mantle. It follows from the above considerations that probably the second conclusion applies, too.

On the basis of the previous considerations, let us now compute a model for intermittent convection in the lower mantle, the conception of which is related to that proposed by Walzer (1974). The model does not depend strongly on the special material constants some of which, by nature, are uncertain. Let us assume that the lower mantle has a somewhat higher zero-pressure density than the upper mantle and that it possesses only a low radioactive heat-generation rate as a result of differentiation. Even if the differentiation of the continents must have taken place at depths corresponding to the present upper mantle (Safronov, 1969; Ringwood, 1975), because the temperature curve there was (and is) closest to the melting curve, the resultant earth-wide instability, nevertheless, would cause that the differentiate of a higher specific weight (at equal p-T conditions) sinks downward and forms the lower mantle there (e.g. Vitjazev, 1980). It is insignificant for our further considerations whether this assumption about the creation of the lower mantle is correct. In any case, like Stacey (1969, p.256), we assume that, at present, the lower mantle has a low rate of heat generation per unit mass of 0.27 erg/g.y. Consequently, the rate of heat generation per unit mass of the upper mantle must be higher so as to obtain the heat flux measured at the surface of the Earth. Incidentally, it also follows from this that the upper mantle is continuously flowing convectively.

To get now somewhat more independent of the special figure assumptions, we shall use three competing models for the lower mantle: In model 1, we assume that the rate of heat generation per unit volume Q^* is constant in time, while it decreases exponentially in the other two models according to the law of radioactive decay:

$$Q^* = Q_0^* \exp(-t^*/t_0^*) \tag{1}$$

where t^* is the time. The time constant varies according to the assumed composition of the radioactive elements. According to McKenzie and Weiss (1975), we have $t_0^* = 2219 \times 10^6$ years for a chondritic composition, whereas $t_0^* = 3248 \times 10^6$ years for a composition after Wasserburg *et al.* (1964). This is to apply for our models 2 and 3, respectively, too. Q_0^* is so defined for models 2 and 3 that the rate of heat generation per unit mass has dropped to 0.27 erg/g.y., 4.5×10^9 years after accretion of the Earth. Thus:

Model	Q_0^* in erg/cm^3.s	t_0^* in Ma
1	4.2864×10^{-8}	$\to \infty$
2	3.2569×10^{-7}	2219
3	1.7131×10^{-7}	3248

In the computation, a mean density of the lower mantle of $\rho_l = 5.01$ g/cm^3 computed after Dziewonski *et al.* (1975) was used. According to the same paper, we assume that the upper surface of the lower mantle lies at a depth of 670 km and the lower surface at 2885.3 km. The heat flux through the lower surface is assumed to be so small that it can be neglected. But this is not an essential pre-supposition of the model. The heat flux through the upper surface of the lower mantle is to be generally expressed by the Nusselt number N of the convection continuously taking place in the upper mantle.

The essential point here is the exponential dependence of viscosity on temperature which applies to both the upper and lower mantles. Let the temperature T_2^* at the surface of the Earth be fixed at 300 K, the temperature T_1^* and T_0^* at the upper and lower surfaces of the lower mantle are functions of time t^*.

Now, an intermediate consideration follows the aim of which is to express the heat flux density χ_σ^* at the upper surface of the lower mantle as a function of T_1^* and T_0^*. For convection in the upper mantle, we assume

(2) $$N = c\ R^{1/3}$$

where R is the Rayleigh number.

(3) $$R = \frac{g\ \alpha\ \Delta T^*\ h_1^3}{k\nu}$$

According to Turner (1973), (2) holds, because it is the only form of the equation that does not depend on the layer thickness h_1. Constant c can still depend on the boundary conditions. As is well known, the Nusselt number N is the ratio of total heat flux to purely diffusive heat flux. Thus, $N = 1$ must hold for $R \leqslant R_c$, with R_c being the critical Rayleigh number. If the oceanic lithosphere is included in the cells of upper-mantle convection, we may count with a stress-free surface and a rigid lower boundary. According to Chandrasekhar (1961, p.42), the critical Rayleigh number for the rigid-free case is $R_c = 1100.65$. From this and from (2), we obtain $c = 0.096\ 861\ 6$. This theoretical value for c lies close to that of Kraichnan (1962) who obtained $N = c\ R^{1/3}$ with $c = 0.089$ for high Prandtl numbers. Turcotte and Oxburgh (1969) and Turcotte *et al.* (1973) use the following approximation for the viscosity of the mantle:

(4) $$\eta = 2.76 \times 10^3 T^* \exp\frac{5.222\times 10^4 + 1.087\times 10^{-7} p^*}{T^*}$$

where T^* is given in K, and p^* in dyn/cm^2. We simplify the formula for the upper mantle by substituting the mean pressure $P_{\mathrm{av}}^* = 1.113 \times 10^{11}$ dyn/cm^2 for p^*, and the mean temperature $T_{\mathrm{av}}^* = (T_1^* + T_2^*)/2$ for T^* in the exponential function. T_{av}^* is a variable with respect to time. Factor T^* in (4) and ΔT^* in (3) are to approximately compensate one another, so that we can use the approximation

(5) $$R = R_k\ \exp(-k_4/2T_{\mathrm{av}}^*)$$

for the upper mantle, with $k_4 = 2 \times 6.432 \times 10^4$ K and $R_k = 5.0351 \times 10^{20}$. R_k was so determined that, according to Tozer (1967), $R = 10^6$ now holds in the upper mantle. Though McKenzie and Weiss (1975) assume $R = 10^5$ to 10^6 for the upper mantle, a rough computation is sufficient for our determination of the Nusselt number. In this computation, we assumed for the upper mantle a temperature of 1900 K in a depth of 335 km. The purely diffusive heat flux density at the interface between the lower and upper mantles is $-\kappa dT^*/dx_3^*$, where κ is the heat conductivity, x_3^* the upward-directed space coordinate. The thermal diffusivity k is $k = \kappa/\rho c_p$. According to McKenzie and Weiss (1975), we use $k = 8 \times 10^{-3}$ cm^2/s for the thermal diffusivity, $\alpha = 2 \times 10^{-5}$ K^{-1} for the thermal expansion coefficient, $c_p = 1.2 \times 10^7$ erg/g.K for the specific heat at a constant pressure. Thus, the total heat flux density at the upper surface of the lower mantle can be expressed by

(6) $$\chi_\sigma^* = k\rho\ c_p \frac{T_1^* - T_2^*}{h_1} N$$

where h_1 is the thickness of the upper mantle. From this and from (2) and (5) we obtain

(7) $$\chi_\sigma^* = k\rho\ c_p\ c\ \frac{T_1^* - T_2^*}{h_1}\ \{R_k\ \exp[-k_4/(T_1^* + T_2^*)]\}^{1/3}.$$

THE COMPUTATION OF INTERMITTENT CONVECTION IN THE LOWER MANTLE OF THE EARTH

Strictly taken, the following set of equations would have to be solved for the lower mantle

(8) $$\rho\left(\frac{\partial \vec{v}^*}{\partial t^*} + \vec{v}^* \cdot \nabla^*\vec{v}^*\right) = -\rho g\vec{k} - \nabla^* p^* + k_1 \frac{\partial}{\partial x_k^*}\left[T^* e^{k_2 T_m(p^*)/T^*}\left(\frac{\partial v_l^*}{\partial x_k^*} + \frac{\partial v_k^*}{\partial x_l^*}\right)\right],$$

(9) $$\nabla^* \cdot \vec{v}^* = 0,$$

(10) $$\frac{\partial T^*}{\partial t^*} + \vec{v}^*\cdot\nabla^* T^* = k\ \nabla^{*2}\ T^* + Q^*(t^*, x_3^*)/\rho_k c_p,$$

(11) $$\rho = \rho_k[1 - \alpha(T^* - T_k)].$$

By *, we denote here the dimensional quantities which are subsequently replaced by dimensionless quantities, with the asterisk being eliminated. Besides the quantities already introduced, we have: ρ = density, $\vec{v}^* = v_k^*$ = velocity vector, $\vec{r}^* = x_k^*$ = location vector, $\vec{k}$ = upward pointing unit vector, g = amount of gravity acceleration; k_1, k_2, ρ_k, T_k are constants, h_0 is the thickness of the lower mantle. The boundary conditions that have already been defined in regard of the heat flux densities must be complemented by

(12) $$v_3 = \frac{\partial v_1}{\partial x_3} = \frac{\partial v_2}{\partial x_3} = \frac{\partial^2 v_3}{\partial x_3^2} = 0$$

for $x_3 = 0$ and $x_3 = h_0$

because one may assume quasi stress-free boundaries for the lower mantle because of its high viscosity as compared to the upper mantle and the outer core (cf. Chandrasekhar, 1961, p.22). The upward pointing component x_3 of the location vector is denoted by z in its dimensionless form; $z = 0$ is assumed to be located in the core-mantle boundary.

We now introduce dimensionless variables.

(13) $$t^* = t\,\frac{h_0^2}{k};\quad \vec{r}^* = \vec{r}h_0;\quad T^* = T\,\frac{kk_1}{g\ \alpha\ h_0^3\ \rho_k};$$

$$p^* = p\,\frac{kk_1}{h_0^2};\quad \vec{v}^* = \vec{v}\,\frac{k}{h_0};\quad \frac{g\ \alpha\ Q^*(t^*, x_3^*)h_0^5}{k^2 k_1 c_p} = Q(t);$$

Let us now consider the static case up to the start of the overturn, i.e. the convection episode. We then obtain from (10) in dimensionless variables the following partial differential equation

(14)
$$\frac{\partial T}{\partial t} - \frac{\partial^2 T}{\partial z^2} = Q(t,z).$$

$Q(t,z)$ is a given function.

According to model 1, we have $Q = 6.445\,96 \times 10^{11}$; according to model 2, we have $Q_0 = 4.897\,8 \times 10^{12}$; according to model 3, we have $Q_0 = 2.576\,2 \times 10^{12}$; (1) with (13) having to be taken into account for the last two models. $t = 0$ defines the time at which development commenced. In the model, it is equated with the accretion of the Earth. t_σ defines the onset of the σ-th convection episode. At first, the temperature $T(z,t)$ is computed in interval $0 < z < 1$ for the first convectionless interval $0 < t \leqslant t_1$. The end of the interval t_1 (or, generally, t_σ) is determined by the surpassing of the critical Rayleigh number in the lower mantle, i.e. by the evolution of the temperature curve during the computation, as will be discussed in greater detail below. In the computation, the initial condition

(15)
$$T(z,0) = 0 \text{ in } 0 < z < 1$$

and the boundary conditions

(16)
$$\frac{\partial T(0,t)}{\partial z} = \psi \quad \text{in } 0 < t < t_1,$$

(17)
$$\frac{\partial T(1,t)}{\partial z} = \chi_\sigma[T(1,t)] \quad \text{in } 0 < t < t_1$$

must be satisfied. In the computations, $\psi \equiv 0$ was used. The boundary condition (17) is non-linear, because from (7) we obtain

(18)
$$\chi_\sigma = k_7[T_2 - T_u^{(\sigma)} - T(1,t)]\left(\exp\frac{-k_8}{T_2 + T_u^{(\sigma)} + T(1,t)}\right)^{1/3}$$

The constants amount to $k_7 = 2.547\,7371 \times 10^6$ and $k_8 = 1.8959 \times 10^{13}$, with the last digits making, of course, no longer any physical sense. They were, however, taken into account for the sake of the accuracy of the mathematical system. $T_u^{(\sigma)}$ is a dimensionless temperature which is newly computed for each $\sigma > 1$, i.e. for each convectionless interval after the first convection episode, according to the results of the preceding processes, while $T_u^{(1)}$ is given as a constant initial temperature.

The set of equations (14) to (18) can be substituted by (18) to (21), the last equation being a non-linear Hammerstein integral equation which facilitates the numerical computation:

(19)
$$T(z,t) = \int_0^t\int_0^1 G(z,\zeta,t-\tau)\,Q(\tau,\zeta)\,d\zeta d\tau - \psi\int_0^t K(z,t-\tau)d\tau + \int_0^t K(1-z,t-\tau)\chi_\sigma[T_1(\tau)]d\tau\ ;$$

(20)
$$T_0(t) = \int_0^t\int_0^1 K(\zeta,t-\tau)\,Q(\tau,\zeta)d\zeta d\tau - \psi\int_0^t K(0,t-\tau)d\tau + \int_0^t K(1,t-\tau)\chi_\sigma[T_1(\tau)]d\tau\ ;$$

(21)
$$T_1(t) = \int_0^t\int_0^1 K(1-\zeta,t-\tau)\,Q(\tau,\zeta)d\zeta d\tau - \psi\int_0^t K(1,t-\tau)d\tau + \int_0^t K(0,t-\tau)\chi_\sigma[T_1(\tau)]d\tau\ .$$

with (22)
$$T_0(t) \equiv T(0,t) \quad \text{and} \quad T_1(t) \equiv T(1,t)\ .$$

G denotes Green's function.

(23)
$$G = G(z,\zeta,t) = \tfrac{1}{2}\left(\theta\left(\tfrac{z-\zeta}{2},L\right) + \theta\left(\tfrac{z+\zeta}{2},L\right)\right),$$

where θ designates Jacobi's theta function:

(24)
$$\theta(s,L) = 1 + 2\sum_{n=1}^{\infty} L^{n^2}\cos 2n\pi s$$

in $0 \leqslant s \leqslant 1$ and $0 \leqslant L < 1$.

(25)
$$L = L(t) = \exp(-\pi^2 t).$$

The kernel is defined by

(26)
$$K(z,t) = \theta(s,L)$$

where $s = z/2$. Interesting considerations about non-linear control processes will be found in von Wolfersdorf (1975). Our main problem now is to solve (21). Once we have found $T_1(t)$, it is possible to determine $T(z,t)$ and $T_0(t)$ by means of (19) and (20). T_1, T and T_0 have been computed for equidistant steps Δt. After each step, the Rayleigh number R in which allowance was made for the temperature dependence of the viscosity in the lower mantle has been determined:

(27)
$$R = \beta\,\exp\left(\overline{k_{15}(z)/[T(z) + T_u^{(\sigma)}]}\right)$$

where $\beta = |\overline{dT/dz}|$. Bars signify averaging over the interval $0 \leqslant z \leqslant 1$, i.e. over the entire lower mantle. k_{15} is defined according to the preceding section and according to the melting temperature curve T_m^* by Stacey (1977). Let us now assume that $T_m^* = 2776$ K, $\eta = 3 \times 10^{26}$ poise and $k_{15}(0.5) = -3.5441 \times 10^{12}$ for radius of 4600 km (i.e. approximately in the centre of our layer). We check for each step whether

(28)
$$R \geqslant R_c$$

has been reached, i.e. whether convection has already started. We used $R_c = 657.5$, because

TABLE I

Model	$T_u^{(1)*}$/K	σ	1	2	3	4	5
		$T_1^*(t_\sigma)$/K	1879.279	1902.670	1920.338	1932.095	1940.885
1	1841	t_σ^*/Ma	1767.065	2802.250	3585.310	4109.274	4502.174
		$(t_\sigma^*-t_{\sigma-1}^*)$/Ma	1767.065	1035.186	783.060	523.964	392.900
		$T_1^*(t_\sigma)$/K	1893.068	1922.992	1936.561	1945.026	
2	1618	t_σ^*/Ma	2894.117	3652.610	4108.232	4449.622	
		$(t_\sigma^*-t_{\sigma-1}^*)$/Ma	2894.117	758.493	455.622	341.390	
		$T_1^*(t_\sigma)$/K	1893.153	1920.408	1933.917	1942.646	
3	1725	t_σ^*/Ma	2819.967	3633.377	4128.443	4496.241	
		$(t_\sigma^*-t_{\sigma-1}^*)$/Ma	2819.967	813.410	495.067	367.797	

the boundaries of the lower mantle are (nearly) stress-free on both the upper and lower surfaces. Then, a thermal compensation through convection is to take place. The initial temperature of the next convection-free interval is computed by means of

(29)

$$T_u^{(\sigma+1)} = \int_0^1 T(z,t_\sigma)\,dz + T_u^{(\sigma)}$$

i.e. a mean value is formed. Index σ increases by one from interval to interval.

COMPARISON OF THE NUMERICAL RESULTS WITH OBSERVED DATA: DISCUSSION

Table I shows the most important numerically computed parameters of the three models. The question, now, is whether these theoretical results can be verified through observed values. Gastil (1960) found that the numerous radioactive age determinations mainly carried out on granites are by no means uniformly, or but slightly, scattered about the time axis. There are distinct maxima at irregular intervals (see Figure 2). This rise and fall of magmatism is considerably slower than the episodic rise and fall which is linked as a synorogeneous magmatism with the controversial orogenetic phases after Stille. The dispute of the geologists over this temporally much shorter phenomenon does not involve the Gastil curve which proves – also if we include more recent age determinations (see Kölbel, 1971) – to be independent of the continent studied. This curve, thus, cannot be explained through local phenomena (e.g. in the lithosphere), we rather have to do here with a global phenomenon. The assumption suggests itself that the curve also defines the rise and fall of the mean heat flux radiated into space. But this is the decisive quantity of the thermal and tectonic history of the Earth. For these reasons, the basic mechanism has to involve large parts of the Earth and must also rank first in energetic respect. Since the lower mantle is *that* part of the Earth having the greatest mass and, in addition, due to the viscosity that increases downward, is unlikely to be *always* flowing convectively, it would be logical to look there for a feed-back mechanism which accounts for the observed slow rise and fall of the global magmatic activity.

Contrary to other mechanisms, the mechanism described here and computed theoretically and numerically shows for the first time a *quantitative* agreement with the most important maxima of the Gastil curve.

In Figures 2 to 4, the convection episodes (overturns) according to models 1 to 3 are compared with the Gastil curve and with the glacial epochs. Model 1 has an unrealistic feature: in this model, the rate of heat generation per unit volume is constant. It is,

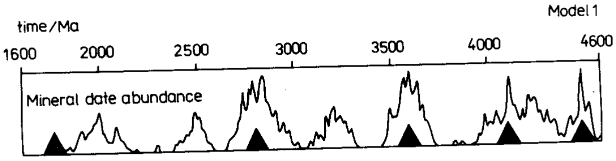

FIGURE 2
Comparison of the temporal distribution of the mineral date abundance after Gastil (1960) with the convection episodes of Model 1.

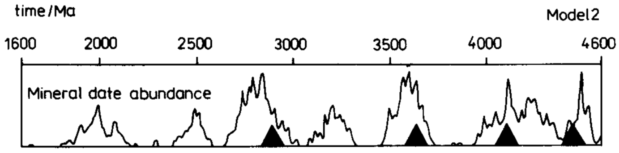

FIGURE 3
Comparison of the temporal distribution of the mineral date abundance after Gastil (1960) with the convection episodes of Model 2.

therefore, no wonder that the first overturn is not accompanied by a maximum in the magmatic activity. Model 2 is more realistic, though the figures assumed there seem to be still too far away from the actual ones (see Figure 3). The best model is model 3, which is derived from the Wasserberg model. Here (see Figure 4), each of the four highest maxima in the magmatic activity curve is exactly accompanied by a convection episode of the lower mantle, the time of the episode being in good agreement with the observed time. This result is all the more remarkable as the distances between the overturns differ from one another. The smaller maxima are indicative of an additional mechanism with a lower energy. We hope that an essential aspect of the development of the Earth has been revealed through model 3.

Finally, some words will be said about the development of ideas that are related to that considered here. It was probably Joly (1930) who for the first time suggested that there is a relationship between the episodicity of orogeny and the radioactivity of the Earth. He, however, grossly overestimated the content of radioactive elements. Tikhonov *et al.* (1970) suggested a periodical melting of the upper mantle.

The idea to explain the curve of magmatic activity of Gastil (1960) and Kölbel (1971) through intermittent subsolidus convection without melting in the lower mantle was first suggested by Walzer (1974). There, too, it was already assumed that the lattice conductivity is too low to conduct the heat outward. The damming of the energy released through the radioactive decay leads, because of the temperature dependence of viscosity, to a gradual increase of the Rayleigh number and to an episodic dissipation of the heat through convection. The convection is episodic, because the low rate of heat generation per unit volume of the lower mantle does not

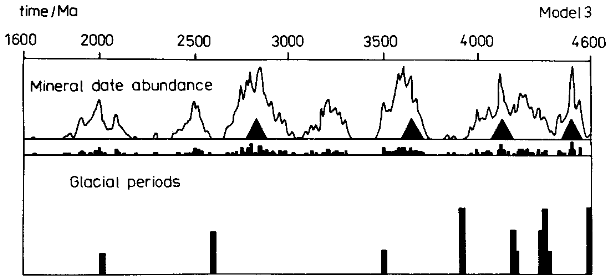

FIGURE 4
Comparison of the temporal distribution of the mineral date abundance after Gastil (1960) and of the glacial periods after Brinkmann (1977) with the convection episodes of Model 3. This model is probably the best one, also with respect to the assumptions regarding heat source distribution.

suffice for a continuous flow. A quantitative proof of the then hypothesis could be given only here. Tozer (1974), also assumed a certain instationarity. He comes to the conclusion that convection in planetary bodies is a block-type rotation with thin sliding faces at the boundaries of the cells and that the necessary heat dissipation from the interior of these cells does not leave the flow system stationary. This conception seems to be realistic to us, because current rolls in the Earth's mantle are probable also for reasons other than those mentioned by Tozer (see Walzer, 1973), and in addition, the possibility suggests itself here to explain phenomena occurring simultaneously earthwide, such as individual orogenies. Qualitative considerations regarding the instationarity of the endogenous forces are found in Rice and Fairbridge (1975).

THE INFLUENCE OF CONVECTION IN THE LOWER MANTLE ON THE GEODYNAMO

Figure 4 shows the agreement of the four main maxima of the granite ages with the four convection episodes in the lower mantle according to model 3. It seems to be reasonable to assume that the intermittent lower-mantle convection also affects movements of the lithosphere, e.g. epeirogeny, through an increase in the flows in the upper mantle. This is actually confirmed. Sloss (1964) finds three very large transgressions for North America in the Phanerozoic, the first and third one being synchronous with the two Phanerozoic convection periods.

As is well known, the lower mantle forms the vessel which contains a much less viscous fluid ($\eta < 10^9$ poise) with metallic electric conductivity. Nowadays, we have reason to believe that the magnetic field of the Earth is essentially generated by flows in the outer core. Since the solutions of the hydromagnetic basic equations are strongly influenced by the boundary conditions, it may be expected that a convection episode in the lower mantle is also reflected in the magnetic field of the Earth. Walzer (1978b), by comparing the curves of other authors, actually found that minima in the frequency of reversals of the geomagnetic dipole correspond to the three big North American transgressions of the Phanerozoic. A similarly close correlation was found for the sediments of the Eastern European platform (see Figure 5). Figure 6 shows a comparison of the transgressions in Eastern Europe with the proportion of reversed polarity of the magnetic dipole of the Earth. Figure 5 shows clearly that in periods of earth-wide transgressions the magnetic polarity changes only infrequently. These periods are also characterized by strong magmatism and by an increased orogenetic activity. As mentioned, the greatest of these activity periods are correlated with convection episodes in the lower mantle. Here, an assump-

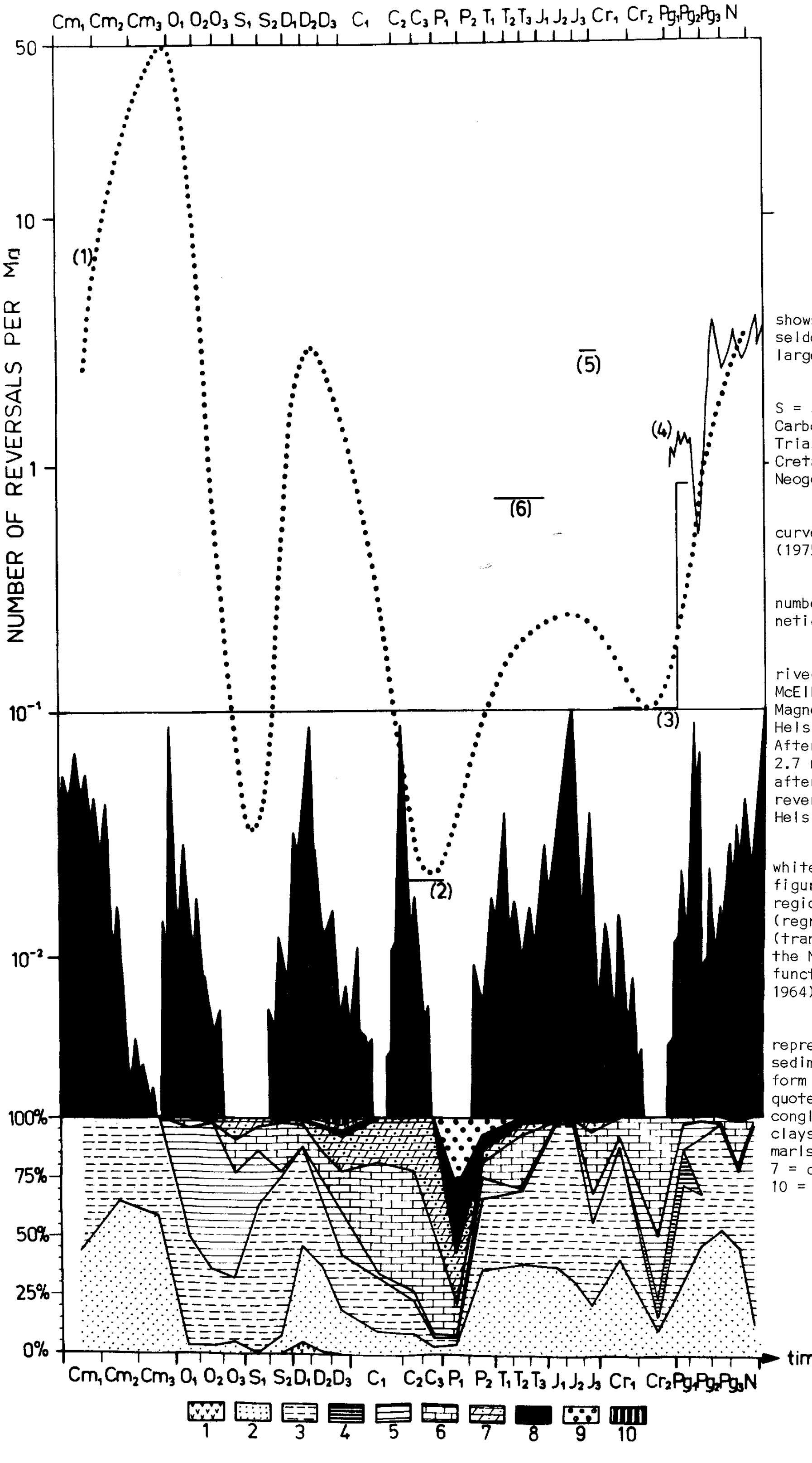

FIGURE 5

The comparison of the curves shows that the geomagnetic dipole seldom changed its polarity during large transgressions.

Cm = Cambrian, O = Ordovician, S = Silurian, D = Devonian, C = Carboniferous, P = Permian, T = Triassic, J = Jurassic, Cr = Cretaceous, Pg = Palaeogene, N = Neogene.

The representation of the curves 1 to 4 stems from Jacobs' (1975) Fig. 4.10.

The curves 1 to 6 show the number of reversals of the geomagnetic dipole as a function of time.

The dotted curve (1) was derived by Jacobs from the work of McElhinny (1971). (2) = Kiaman Magnetic Interval. (3) = After Helsley and Steiner (1969). (4) = After Heirtzler *et al.* (1968). (5) = 2.7 reversals/ma in the late Jurassic after Vogt *et al.* (1972). (6) = 0.7 reversals/ma in the Triassic after Helsley (1972).

The distribution of black and white in the central part of the figure shows the percentage of regions of non-depositional hiatus (regressions) and of depositions (transgressions), respectively, on the North American platform as a function of time (after Sloss, 1964).

The lower part of the figure represents the distribution of sediments on the East European platform (after Ronov *et al.* (1969), quoted after Beloussov (1972)). 1 = conglomerates; 2 = sandstones; 3 = clays; 4 = siliceous rocks; 5 = marls; 6 = limestones and chalk; 7 = dolomites; 8 = gypsum; 9 = salt; 10 = effusive rocks and tuffs.

tion (Walzer, 1978b) is to be put forward as to why the geomagnetic polarity changes so infrequently just then. When the lower mantle is in a state of rest, the isothermal faces near the core-mantle interface are probably almost spherically symmetric. Therefore, the magnetic field should have a high degree of symmetry (e.g. rotational symmetry). The generalized Cowling theorems (anti-dynamo theorems according to Jacobs, 1975, pp.129-130) show that fields approximating certain symmetries tend to be unstable. In the episodes of lower-mantle convection and subsequently, however, the temperature field in the lower mantle most probably deviates more from the spherical symmetry, so that the flow velocity field in the outer core would also be less symmetric, resulting in a more stable magnetic field. The question of the inversion mechanism of the magnetic field is still largely unsolved. A possibility for finding a solution to it is perhaps indicated by the preceding geophysical observations.

The conclusions of this work are to be found in the abstract.

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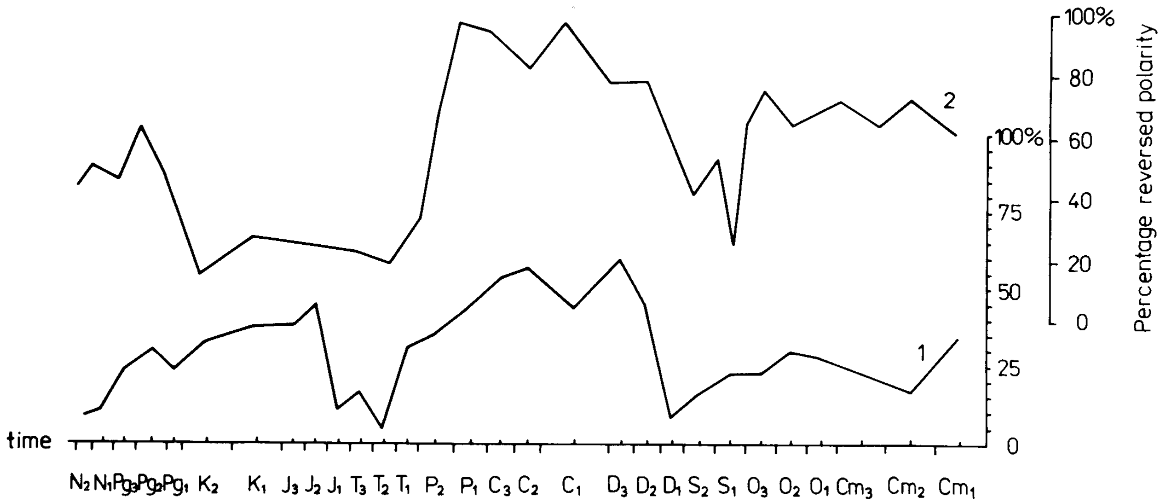

FIGURE 6

Curve 1 shows the variations of the sedimentation area of the East European platform (in percent of the whole area) after Ronov (1961), Sholpo (1969) and Beloussov (1972). It is interesting to compare curve 1 with curve 2 which represents the percentage of reversed polarity of the geomagnetic dipole (after Creer, 1975, and Whyte, 1977).

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